# Dynamic pricing with strategic customers 

## 21/09/2012

Dr. Jochen Gönsch • Prof. Dr. Robert Klein ( $\boxtimes$ ) • Michael Neugebauer • Dr. Claudius Steinhardt<br>Department of Analytics \& Optimization, University of Augsburg Augsburg, 86135 Augsburg, Germany<br>Email: robert.klein@wiwi.uni-augsburg.de<br>Dr. Jochen Gönsch<br>Email: jochen.goensch@wiwi.uni-augsburg.de<br>Michael Neugebauer<br>Email: michael.neugebauer@wiwi.uni-augsburg.de<br>Dr. Claudius Steinhardt<br>Email: claudius.steinhardt@ wiwi.uni-augsburg.de


#### Abstract

This paper provides an overview of the literature on dynamic pricing with strategic customers. In the past, research on dynamic pricing was mostly concerned with optimally pricing products over time in a market with myopic customers. In recent years, the consideration of strategic customers, who can delay a purchase to take advantage of a future discount, has dramatically increased. This paper's main contribution is the development of a comprehensive classification scheme to structure the field of research and, based upon this, a systematic overview of all relevant papers. We then present in detail the various aspects considered in the literature together with their motivation from industry and state the major findings of the most relevant papers. Further attention is given to important problem extensions proposed in the literature that have been considered in only a few papers and are usually motivated by specific practical applications. Finally, promising directions for future research are indicated.


Keywords: Dynamic Pricing, Strategic Customers, Optimization
JEL Classification: M19

## 1 Introduction

For years, product prices in the retail and service industry were adjusted only very rarely over time. The reasons for this were mainly associated with the high costs involved in price changes. These costs essentially arose from extensive manual effort and huge levels of investment required for the purchase of hardware and software to implement automatic price adjustment systems. In addition there was insufficient knowledge regarding actual customer behaviour in the event of price changes in terms of the propensity to purchase (e.g. Elmaghraby/Keskinocak 2003, p. 1288). The now ubiquitous use of the Internet as a major sales channel as well as the continued development of IT hardware have both fundamentally changed this situation (e.g. Diller 2008, p. 443 et seq.). The transaction costs of price changes have been reduced to a minimum (e.g. Elmaghraby et al. 2008, p. 126). Sellers are now able to implement automated pricing mechanisms in place of traditional single fixed posted prices without having to incur significant costs (e.g. Simon/Faßnacht 2009, p. 508). Furthermore, information on customer behaviour can more easily be collected on a broad and continuous basis, be analysed systematically, and then automatically be incorporated in future pricing decisions.

It is therefore no real surprise that a large number of scientific papers have appeared over the last two decades which deal with the concept of so-called dynamic pricing ${ }^{1}$. Dynamic pricing aims at maximizing profits by dynamically changing offer prices within the selling horizon in order to optimally exploit changes in demand or competition-related conditions. General review articles on the subject of dynamic pricing have been written by Bitran/Caldentey (2003), Elmaghraby/Keskinocak (2003), Chan et al. (2004), and Gönsch et al. (2009). In most scientific publications on dynamic pricing, the modelling of customer behaviour traditionally only plays a minor role. Customers are mostly assumed to act in a myopic way, meaning that they immediately make a purchase if the price is below their personal valuation, that is, if they are able to achieve a positive customer surplus by making such purchases. As a result, future price trends have no influence on purchasing decisions; a seller can therefore set his prices without having to fear any potential disadvantages, for example, as a result of likely subsequent reductions in price (e.g. Elmaghraby/Keskinocak 2003, p. 1290).
However, in recent years, researchers have begun to realize that the assumption of such uninformed, myopic customers may be somewhat out of date. Customers are increasingly thinking strategically and include future price trends as part of their calculations. Even if purchases which are made immediately lead to a positive consumer surplus, a strategic customer will nevertheless take into account all future purchase opportunities and delay making a purchase if necessary in order to achieve a higher surplus. For example, if he believes that the seller will lower the price for the desired product in future, then it may make sense, under certain circumstances, for him to wait for the reduction in price to take effect and then make the purchase. This will in turn usually offset the market power of a monopolistic seller to a certain extent (e.g. Besanko/Winston 1990). Thus, a major concern from the seller's perspective is to discourage this strategic waiting. The consideration of strategic customer behaviour is specifically important because customers today are also very much supported in their decisionmaking by modern Internet technologies and software. An example of this is a price ticker which tracks the price trends of certain products on the Internet and documents them for the user (e.g. Simon/Faßnacht 2009, p. 509). Alternatively the customer can set a price alert to his desired price and receive a notification when the price has fallen to the targeted level. In addition, there are data mining programmes which anticipate the behaviour of the various automatic pricing systems and provide a recommendation for the time of purchase (see Etzioni et al. 2003). As a result of these numerous tools, it is less surprising that dynamic pricing approaches, in which the seller fails to recognise the strategic behaviour of customers by mis-
take and therefore sets prices for myopic customers, perform poorly in comparison. The literature reports on losses of profit of between $7 \%$ and $50 \%{ }^{2}$
In this paper, we review the scientific literature on dynamic pricing with strategic customers. To define the scope, we use the following common definitions of "dynamic pricing" and "strategic customer behaviour": We define dynamic pricing as the planned action of a seller to change his posted prices at arbitrary times within the selling horizon (that is, "dynamically") in order to respond to changes in demand or competition-related conditions with the goal of maximizing total profit (e.g. Klein/Steinhardt 2008, p. 176 for a similar definition). ${ }^{3}$ Regarding the term "strategic customer behaviour" ${ }^{4}$, we assume that strategic customers will compare the current purchasing opportunity to potential future opportunities and decide whether to purchase immediately or to wait. That is, customers attempt to "get more for less" by timing a purchase appropriately, for example in anticipation of future price changes (e.g. Levin et al. 2010 for a similar definition). In particular, in some cases customers may know the future prices because the firm publicly announces them. In the remaining cases, strategic customers are able to consistently anticipate the firm's future prices by building rational forward-looking price expectations. Overall, we consider work on settings where the seller has to optimize his offer prices dynamically with the goal of maximizing total profit, while customers are strategically timing their purchase in order to maximize their individual consumer surplus. The main contribution of the scientific literature in this context lies in developing appropriate mathematical optimization models and analytically deriving managerial insights. It originates from the field of Operations Research, at the interface of Operations Management to Marketing and Economics.

Note that with this scope, we explicitly exclude research that is either based on other definitions of "dynamic pricing" or "strategic customer behaviour", as well as research that does not include both aspects in one model simultaneously. In dynamic pricing, for example, there is research on interactive selling mechanisms that are sometimes also denoted "dynamic pricing", for example, auctions ${ }^{5}$, competitive bidding, and one-to-one negotiations (e.g. Bichler et al. 2002). Regarding strategic customer behaviour, there are a number of authors who call the strategic customers we consider "(completely) rational customers" (e.g. Shen/Su 2007 and Levin et al. 2010) who can be opposed to customers that show bounded rational behaviour; this can include different aspects, for example the theory of adoption and diffusion (e.g. Bass 1969), the theory on perceived risk (e.g. Roselius 1971 and Shoemaker/Shoaf 1975), and the reference price concept (e.g. Helson 1964 and Kahneman/Tversky 1979). Although out of scope of this paper due to the different assumptions on customer behaviour, note that there is a body of literature explicitly combining dynamic pricing and bounded rational customer behaviour as described above. This is specifically true for the reference price concept that is particularly popular in the Marketing Science literature dealing with price promotions in the consumer goods' sector (e.g. Pashigan/Bowen 1991, Greenleaf 1995, and Blattberg/Neslin 1990 for an overview). There are a number of approaches characterizing dynamic pricing strategies with reference price effects where customers make their decision to purchase while considering the present and past prices (see Kopalle et al. 1996, Fibich et al. 2003, and Popescu/Wu 2007). However, no (expected) future prices are considered; this makes those customers different from the completely rational (strategic) customers on whom we focus.

Finally, in the context of strategic customer behaviour, there is a stream of Marketing Science literature that does not focus on a mathematical dynamic price optimization like we do, but empirically examines strategic customer behaviour with exogenously given prices that are thus not endogenously chosen optimally by a seller. An example of this direction of research is the estimation of dynamic structural models. For example, Erdem et al. (2003), Sun et al. (2003), and Sun (2005) estimate such models based on scanner panel data for frequently purchased consumer goods and analyse customers' forward-looking price expectations and their
impact on purchase timing. Melnikov (2000), Song/Chintagunta (2003), and Erdem et al. (2005) consider dynamic structural models in the context of high-tech durable goods where the prices fall quickly over time (also see Chintagunta et al. 2006). While the aforementioned papers presume that customers behave strategically, Chevalier/Goolsbee (2009) and Li et al. (2011) are part of very little and ongoing research that empirically provides direct evidence of strategic customers. To the best of our knowledge, there are only very few papers which combine empirical analysis with a dynamic price optimization section in their studies and that are therefore in the scope of our paper, at least to a certain extent (see Nair 2007 and Koenigsberg et al. 2008). Nair (2007) conducts a two-stage analysis: Firstly, he proposes a method to obtain estimates of demand parameters under forward-looking customer behaviour. Secondly, he takes these demand parameters as given and calculates numerically the corresponding firms' optimal pricing policies. Koenigsberg et al. (2008) empirically test the validity of some of their model assumptions before they conduct their game theoretic analysis. For example, from the data they find evidence of the existence of two segments of customers (tourists and business travellers) and incorporate these two segments into their model.

Especially in the Operations Management community, interest in strategic customer behaviour has grown considerably in recent years. Our evaluations have shown that a variety of research papers have been published, specifically since 2006, which, following our above scoping, take into account dynamic pricing with strategic customer behaviour. A high proportion of them have appeared in high quality journals like Management Science, Operations Research, and others; this underlines the relevance of this research topic. Thus, it is highly probable that there will be a large number of additional papers in the coming years. Furthermore, there are surveys written by Shen/Su (2007), Ho/Su (2009), Tang (2010), Zhou/Wu (2011), and Aviv/Vulcano (2012) as well as a collected volume edited by Netessine/Tang (2009). However, $\mathrm{Ho} / \mathrm{Su}$ (2009), Zhou/Wu (2011), and Aviv/Vulcano (2012) do not present a comprehensive review of all relevant research papers, but only selected models. The same is true for Netessine/Tang's (2009) book where the original models' authors present their work in detail in the individual chapters. ${ }^{6}$ The relatively new general review article by Tang (2010) only goes into strategic customer behaviour over two pages. The older paper of Shen/Su (2007) only represents a very small part of today's existing literature, due to the extensive research activities carried out in recent years. None of the papers attempt to classify the available literature in a structured manner, based, e.g. on a set of specific criteria.

Based on these shortcomings, the objective of this paper is to present an updated, comprehensive, and systematic review of the scientific literature on dynamic pricing with strategic customers. The paper is structured as follows: In Section 2, we present the classification scheme developed and provide a structured overview of all relevant papers according to the classification scheme's criteria. We also name specific extensions to the problem setting wherever a paper considers further aspects motivated by specific practical considerations. This already gives the reader an impression of the various aspects that render this field of research very heterogeneous. Section 3 considers each criterion of the classification scheme introduced in Section 2 in depth following a two-fold purpose. First, we explain the meanings of the different criteria in detail along with their motivation from industry. Second, we give an outline of the current status of research using the most relevant literature and summarize the most important publications' insights related to the criterion under consideration. After examining each criterion in isolation, we conclude the section with a brief summary of common combinations of criteria values and relate them to specific industries. In Section 4, we discuss in detail the specific problem extensions named in Section 2. Most papers are characterised by the consideration of such further aspects motivated by practical considerations that are so specific that they cannot be grouped by a general classification. The paper is then completed with a summary and research outlook in Section 5.

## 2 Classification scheme and classification of the literature

Even though restricted to research strictly following the definition of dynamic pricing with strategic customers as given in Section 1, a review of the existing publications still shows that the literature is very heterogeneous with respect to the various model types used. This is mainly because scientists often attempt to illustrate certain aspects of specific practical applications as clearly as possible, as well as derive meaningful results at the same time. In addition, the incorporation of strategic customers is quite complex compared to the traditional assumption of myopic customers; all authors have to make certain restrictive assumptions in order to obtain manageable and analytically tractable models. Finally, the literature is influenced by many different fields, e.g., Marketing, Economics, and Operations Management, each of them having a certain perspective and research focus.

Table 1: Classification Scheme

| Criterion | Abbreviation | Values | Abbreviations | Frequencies |
| :---: | :---: | :---: | :---: | :---: |
| Capacity | C | - Limited | L | 50.0 \% |
|  |  | - Capacity as decision variable | CDV | 45.8 \% |
|  |  | - Availability risk as decision variable | ADV | 8.3\% |
|  |  | - Infinite | I | 27.1 \% |
| Time of pricing | TOP | - At the beginning of the selling horizon | BSH | 60.4 \% |
|  |  | - During the selling horizon | DSH | 58.3 \% |
| Pricing policy | PP | - Markdown | MD | 43.8 \% |
|  |  | - Markup | MU | 2.1 \% |
|  |  | - Markup and markdown possible | MUD | 52.1 \% |
| Demand arrival process | DAP | - Simultaneous at the beginning of the selling horizon | SIM | 62.5 \% |
|  |  | - Sequential during the selling horizon | SEQ | 41.7 \% |
| Number of time periods | NOP | - Two | T | 52.1\% |
|  |  | - Finite (>2) | F | 22.9 \% |
|  |  | - Infinite | 1 | 10.4 \% |
|  |  | - Continuous time | CT | 18.8 \% |
| Time preference | TP | - Customers | C | 31.3 \% |
|  |  | - Sellers and customers | SC | 29.2 \% |
|  |  | - Not considered | NC | 47.9 \% |
| Market type | MT | - Monopoly | M | 89.6 \% |
|  |  | - Oligopoly | O | 14.6\% |
| Information | IS | - Complete information | CI | 43.8 \% |
| Setting |  | - Incomplete information | II | 79.2 \% |

However, it is possible to identify some basic problem and modelling aspects common to all publications under consideration. All papers aim at developing mathematical optimization models that are formulated from a seller's perspective, whose objective function is profit maximization. The decision variables within the models are basically the offer prices that the seller can set for his product throughout a selling horizon. Moreover, in a number of papers, he controls additional variables, mostly related to capacity. The seller is confronted with strategic customers who seek to maximize their individual intertemporal utility. For a given customer, his utility is defined by his consumer surplus, that is, the difference between his individual valuation (willingness to pay) and the price. At every point in time each customer faces the decision whether to buy immediately or to postpone his purchase decision and "speculate" on a higher consumer surplus if he buys later on. From a modelling perspective, these assumptions on customer behaviour are always incorporated in the seller's optimization problem as constraints, although the customers' objective function is sometimes additionally stated explicitly. As described in Section 1, the customers base their buying decision purely on the rational expectations they build about future prices. Therefore, regarding the models in
use, the problem is always analysed in a game theoretic framework. Most of the resulting optimization models are then solved analytically for Nash equilibria ${ }^{7}$.

In order to capture the differences of the publications regarding the problem assumptions, we propose a comprehensive classification scheme that is based on the observation that the literature essentially differentiates in terms of capacity, time of pricing, pricing policy, demand arrival process, number of time periods, time preference, market type, and considered information setting. Table 1 provides an overview of these distinguishing criteria along with the corresponding various values examined in the literature as well as their frequencies of occurrence. Given that certain papers examine several values of a criterion or are unable to categorise them, some amounts differ from $100 \%$. We discuss the different classification criteria along with the relevant literature in detail in Section 3.

On the basis of this classification scheme, we now classify the existing publications in Table 2. If a paper focuses on a specific criterion's value, but an additional value (albeit one which is given less attention) is also examined as part of the paper, then this second value is given between brackets. If a paper cannot be classified with respect to a specific criterion in a meaningful manner, then this is indicated by "--". Furthermore, besides the aforementioned classification criteria, most papers are additionally characterised by the consideration of further aspects that are motivated by specific practical considerations and that can have significant impact on the resulting models and the generated insights. However, they are so specific that it is not reasonable to group them by a general criterion of a classification scheme. Therefore, these additional aspects are listed in the "Extensions" column. The specific problem extensions are presented in detail in Section 4.

Table 2: Classification of the literature

| Reference | C | TOP | PP | DAP | NOP | TP | MT | IS | Extensions |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aviv/Pazgal (2008) | L | $\begin{aligned} & \hline \text { BSH, } \\ & \text { DSH } \end{aligned}$ | MD | SEQ | CT | C | M | II |  |
| Bansal/Maglaras (2009) | $\begin{gathered} \mathrm{L}, \\ \mathrm{ADV} \end{gathered}$ | BSH | MUD | SIM | F | NC | M | CI | Auctions and mechanism design, Customers' and seller's risk preference |
| Besanko/Winston $(1990)$ | I | DSH | MUD | SIM | F, (I) | SC | M | CI |  |
| Bhalla (2012) | I | DSH | MUD | SEQ | T | SC | M | II | Uncertainty in terms of customers' individual valuation |
| $\begin{aligned} & \hline \text { Biyalogorsky } \\ & (2009) \\ & \hline \end{aligned}$ | L | BSH | MU | SEQ | T | NC | M | II | Additional purchase options |
| Borgs et al. (2011) | L | BSH | MUD | SEQ | F | NC | M | CI, II |  |
| Cachon/Swinney $(2009 \mathrm{a})$ | CDV | DSH | MD | SIM | T | C | M | II | Short-term replenishment |
| Cachon/Swinney (2011) | CDV | BSH | MD | SIM | T | C | M | II | Short-term replenishment |
| Chen/Zhang (2009) | I | DSH | MUD | SIM | T | SC | O | CI |  |
| Dasu/Tong (2010) | L | $\begin{gathered} \mathrm{BSH}, \\ (\mathrm{DSH}) \end{gathered}$ | MD | SIM | F | NC | M | $\begin{gathered} \text { II, } \\ \text { (CI) } \\ \hline \end{gathered}$ |  |
| Elmaghraby et al. $(2008)$ | L | BSH | MD | SIM | F | $\begin{gathered} \mathrm{NC}, \\ (\mathrm{C}) \\ \hline \end{gathered}$ | M | CI, II | Demand for multiple product units |
| Elmaghraby et al. (2009) | L | BSH | MD | SEQ | CT | NC | M | $\begin{gathered} \text { II, } \\ (\mathrm{CI}) \end{gathered}$ | Additional purchase options, Auctions and mechanism design |
| Gallego et al. (2008) | $\begin{gathered} \mathrm{L}, \\ \mathrm{ADV} \end{gathered}$ | BSH | MD | SIM | T | $\begin{gathered} \mathrm{C}, \\ \mathrm{SC}) \end{gathered}$ | M | $\begin{aligned} & \begin{array}{l} \text { CI, } \\ \text { (II) } \end{array} \end{aligned}$ |  |
| Gallien (2006) | L | -- | -- | SEQ | CT | SC | M | II | Auctions and mechanism design |


| Gallien/Gupta $(2007)$ | L | BSH | MUD | SEQ | CT | SC | M | II | Auctions and mechanism design |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Huang/van Mieghem (2011) | CDV | $\begin{aligned} & \hline \text { BSH, } \\ & \text { DSH } \end{aligned}$ | MD | SIM | T | C | M | II |  |
| Jerath et al. (2010) | L | DSH | MD | SIM | T | NC | O | CI, II |  |
| $\begin{aligned} & \hline \text { Kim/Swinney } \\ & (2011) \\ & \hline \end{aligned}$ | CDV | $\begin{gathered} \text { BSH, } \\ \text { (DSH) } \end{gathered}$ | MD | SEQ | T | NC | M | $\begin{gathered} \hline \text { II, } \\ \text { (CI) } \\ \hline \end{gathered}$ | Product quality choice |
| Koenigsberg et al. (2008) | L | DSH | MUD | SEQ | F | NC | M | II | Uncertainty in terms of customers' individual valuation |
| Lai et al. (2010) | CDV | DSH | MD | SIM | T | C | M | II | Price matching |
| Levin et al. (2009) | L | DSH | MUD | SIM | F | C | O | II | Uncertainty in terms of customers' individual valuation |
| Levin et al. (2010) | $\begin{gathered} \mathrm{L}, \\ (\mathrm{CDV}) \end{gathered}$ | $\begin{gathered} \mathrm{DSH}, \\ (\mathrm{BSH}) \end{gathered}$ | MUD | SIM | F | C | M | II | Uncertainty in terms of customers' individual valuation |
| Li/Zhang (2012) | CDV | DSH | MUD | SEQ | T | NC | M | II | Price matching, Additional purchase options |
| Liu/Xiao (2008) | $\begin{gathered} \mathrm{L}, \\ (\mathrm{CDV}) \end{gathered}$ | BSH | MD | SIM | T | NC | M | II | Uncertainty in terms of customers' individual valuation |
| $\begin{aligned} & \text { Liu/van Ryzin } \\ & (2008) \\ & \hline \end{aligned}$ | CDV | BSH | MD | SIM | T | $\begin{aligned} & \mathrm{NC}, \\ & (\mathrm{SC}) \end{aligned}$ | M, (O) | CI | Customers' and seller's risk preference |
| Liu/Zhang (2012) | I | DSH | MUD | SIM | F | SC | O | CI |  |
| Mak (2008) | $\begin{gathered} \mathrm{L}, \\ (\mathrm{CDV}) \\ \hline \end{gathered}$ | DSH | MD | SIM | T | SC | M | CI, II |  |
| Mak et al. (2012) | I | DSH | MUD | SIM | F | C | O | II |  |
| Mantin/Granot (2010) | I | DSH | MUD | SIM | T | SC | O, (M) | CI |  |
| $\begin{aligned} & \hline \text { Mersereau/Zhang } \\ & \text { (2012) } \\ & \hline \end{aligned}$ | L | BSH | MD | SIM | T | NC | M | CI, II |  |
| Nair (2007) | I | DSH | MUD | $\begin{aligned} & \hline \text { SEQ, } \\ & \text { (SIM) } \end{aligned}$ | I, (T) | SC | M | $\begin{gathered} \hline \mathrm{II}, \\ \text { (CI) } \\ \hline \end{gathered}$ |  |
| Osadchiy/Vulcano (2010) | CDV | BSH | MD | SEQ | CT | SC | M | II | Additional purchase options |
| Parlatürk (2012) | I, (L) | $\begin{gathered} \text { DSH, } \\ (\mathrm{BSH}) \end{gathered}$ | MUD | SIM | T | C | M | CI | Product quality choice |
| Prasad et al. (2011) | CDV | BSH | MUD | SEQ | F | NC | M | II | Additional purchase options, Uncertainty in terms of customers' individual valuation, Customers' and seller's risk preference |
| Su (2007) | $\begin{gathered} \mathrm{L}, \\ \text { ADV, } \\ \text { (CDV) } \end{gathered}$ | BSH | MUD | SEQ | CT | C | M | CI |  |
| Su (2009) | L | DSH | MUD | SEQ | T | C | M | II | Uncertainty in terms of customers' individual valuation |
| Su (2010a) | $\begin{gathered} \mathrm{L}, \\ (\mathrm{CDV}) \\ \hline \end{gathered}$ | DSH | MUD | SEQ | T | NC | M | II | Speculators |
| Su (2010b) | $\begin{gathered} \mathrm{I}, \\ (\mathrm{CDV}) \end{gathered}$ | DSH | MUD | SIM | I | NC | M | II | Demand for multiple product units |
| Su/Zhang (2008) | CDV | BSH | MD | $\begin{gathered} \text { SIM, } \\ \text { (SEQ) } \end{gathered}$ | T | $\begin{gathered} \text { NC, } \\ \text { (C) } \\ \hline \end{gathered}$ | M | II |  |
| Swinney (2011) | CDV | $\begin{aligned} & \text { BSH, } \\ & \text { DSH } \end{aligned}$ | MUD | SIM | CT | C | M | II | Short-term replenishment, Uncertainty in terms of customers' individual valuation |
| Tereyagoglu/ Veeraraghavan (2012) | CDV | BSH | MD | SIM | T | NC | M | II |  |
| Villas-Boas (2004) | I | $\begin{aligned} & \text { DSH, } \\ & \text { (BSH) } \end{aligned}$ | MUD | SEQ | I | SC | M | CI |  |
| Wang/Hu (2011) | CDV | BSH | MD | SIM | T | NC | M | II | Customers' and seller's risk preference |


| Xu (2011) | I | $\begin{aligned} & \text { DSH, } \\ & \text { (BSH) } \end{aligned}$ | MUD | SIM | I | SC | M | II | Price matching, Uncertainty in terms of customers' individual valuation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Yin et al. (2009) | CDV | BSH | MD | SEQ | CT | NC | M | II |  |
| Yousefi et al. (2011) | L | BSH | MUD | SIM | CT | NC | M | II | Auctions and mechanism design, Customers' and seller's risk preference |
| Yu et al. (2011) | $\begin{gathered} \mathrm{L}, \\ \mathrm{ADV}, \\ \text { (I), } \\ \text { (CDV) } \end{gathered}$ | DSH | MUD | SEQ | T | NC | M | CI, II | Additional purchase options, Uncertainty in terms of customers' individual valuation, Customers' and seller's risk preference |
| Zhang/Cooper (2008) | $\begin{gathered} \text { ADV, } \\ \text { I, L } \\ \hline \end{gathered}$ | $\begin{aligned} & \text { BSH, } \\ & \text { DSH } \end{aligned}$ | MD | SIM | T | NC | M | CI |  |

## 3 Classification criteria and status of research

The purpose of this section is two-fold. First, we explain the meanings of the different criteria and corresponding values of the classification scheme introduced in Section 2. Second, we give an outline of the current status of research using the most relevant literature.

Each of the Sections 3.1 to 3.8 focuses on one classification criterion. For this criterion, the various values examined in the literature along with their motivation from practical examples are discussed. In every subsection, we present some selected publications in detail, for which the respective criterion's value represents one of the key drivers of the publications' insights. In this context, it would of course be desirable to extract major results and general insights that are directly driven by a certain classification criterion. Wherever possible, we state such results. However, in order to avoid "comparing apples to oranges", direct comparisons of one paper's results to another's can be made only in case of "ceteris paribus", that is, if the exact problem formulation considered differs only with regard to one criterion and all other criteria's values are identical and no specific extensions are modelled. However, given the heterogeneity of the area, it turns out that this is prevalently only true if one publication successively examines different values of the respective criterion. If the problem formulation differs in more than one criterion or specific extensions are considered, we simply state the authors' major findings regarding the criterion under consideration. Quite often it shows that a criterion's value can drive results in one direction given a certain problem setting as defined by the other criteria's values and lead to completely different results in another problem setting. After having examined each criterion in isolation, Section 3.9 gives a brief summary of common combinations of criteria's values and relates them to specific industries.

### 3.1 Capacity

Quite early research papers - which, from today's perspective, can be seen to belong to the field in the broadest sense - mainly examine problems with infinite capacity, such as papers from the Economics literature (e.g. Coase 1972, Stokey 1979, and Stokey 1981). Besanko/Winston (1990) is known to be the first publication on dynamic pricing with strategic customers, considering infinite capacity as well. In this paper, the authors show that falling prices are the optimum solution in the face of a strategic buyership over time. Quite recently, Mantin/Granot (2010) have also considered the issue of infinite capacity.

However, in the recent literature approaches with limited capacity are the norm (e.g. Aviv/Pazgal 2008, Elmaghraby et al. 2008, and Jerath et al. 2010). In these models, it can happen that a customer does not obtain a product despite having the intention to buy it (availability risk) as it is already sold out due to the (stochastic) demand of other customers, or be-
cause remaining inventory is insufficient to serve all customers with purchase intentions and the seller therefore allocates the inventory in a random manner. As a result, this reduces the expected benefit of a purchase in the future; purchases tend to be made earlier than in cases where capacity is infinite. It is important to note that, although it is assumed that the availability risk may arise for the customer and may also be included by all market participants in their rational considerations, the aforementioned papers assume that the seller does not make any effort to exacerbate the shortage and take advantage of the situation for his own benefit.

Nevertheless, a significant number of papers takes into consideration that the shortage in capacity could intentionally be influenced by the seller by modelling the initial capacity as a decision variable, alongside the price. By choosing the total capacity available for sale at the beginning of the selling horizon, the seller is able to directly control the availability of the product and thus indirectly control the availability risk for customers. If this risk increases, then it becomes increasingly less attractive for strategic customers to postpone a purchase, whereas at the same time it may lead to the seller losing sales due to the lower initial capacity. Liu/van Ryzin (2008) present such an approach that includes capacity decisions and point out that it depends on the proportion of customers with a high valuation whether or not an availability risk should be generated by means of a low initial capacity. One major result they get from their model is that capacity decisions are far more important than any pricing decisions which are examined as well. Cachon/Swinney (2009a) also consider capacity to be a decision variable and find that the availability risk is particularly important if all prices are announced at the beginning of the selling horizon, meaning that there is no uncertainty for customers regarding future prices (price risk). Another insight they gain is that monopolists in a market with strategic customers generally select a smaller initial capacity, make smaller price cuts, and earn less than they would in a market consisting solely of myopic customers. Su (2007), Mak (2008), Levin et al. (2010), and Su (2010a) also point out potential extensions to their models in terms of capacity decisions. In the technical report of their paper, Huang/van Mieghem (2011) focus on a particular aspect; they consider a manufacturer of industrial products who obtains advanced demand information from clickstreams on his website and uses this information to decide on initial capacity. They also analyse the impact on the behaviour of strategic customers if they are aware that this analysis is taking place.

Some papers directly consider the customers' availability risk as decision variable of the seller in addition to the price. In this context, the seller is usually able to specify the proportion of customers that he will serve at each point in time, as for example in Bansal/Maglaras (2009). Zhang/Cooper (2008) compare the importance of pricing decisions versus availability decisions. They show that if capacity restrictions are irrelevant and prices can be set optimally, increasing the availability risk (rationing) cannot improve revenue. However, when prices are (suboptimally) fixed in advance, rationing can improve revenue. Although considerably improving revenue compared to unrestricted availability, rationing only enables the seller to obtain a small fraction of the optimal revenue when prices are set optimally without rationing. In some models, the seller specifies maximum contingents to offer during some intervals of the selling horizon instead of the proportion of customers to serve. An example is Yu et al. (2011), where the seller decides on a maximum contingent for advance selling. They show that for large capacity, advance selling should be unlimited, limited for medium capacity, and not used at all for relatively low capacity.

The implicit assumption underlying this subsection so far has been that customers value availability positively, because they can only enjoy the product if they are able to buy it. Tereyagoglu/Veeraraghavan (2012) consider the exact opposite. So-called conspicuous customers value more highly the products that are hard to find. This might apply to luxury products and innovative gadgets that bring some exclusivity to their owner. The authors show that the firm may nonetheless offer a high availability of products. But there are also conditions under
which scarcity would improve profits. To credibly commit to such a strategy, the firm could seek to increase its production cost by using an expensive supplier or raw materials.

### 3.2 Time of pricing

The next criterion of the classification scheme relates to the point in time when the seller decides on the prices to set. This can either be before or during the selling horizon. When the seller decides at the beginning of the selling horizon, he publishes all prices upfront. In this context, the department store chain Filene's Basement is often cited as a particularly vivid example. By using its "Automatic Mark Down System", the company reduces all of its prices by $25 \% 12$ days after the start of a sale, by half 18 days after the start of a sale, and by $75 \%$ 24 days after the start of a sale. If a product remains unsold for 30 days, it will then be donated to charity - something of which all market participants are aware (see Filene's Basement 2012). Also well-known are tour operators' printed catalogues which are usually valid for half a year and include a price list with detailed information on early-bird discounts. If the seller can credibly commit to such a price path and customers trust the fact that the seller will not deviate further from this price path, they include these prices directly in their purchasing decisions (e.g. Liu/van Ryzin 2008, Elmaghraby et al. 2009, and Yin et al. 2009). However, this commitment prevents the seller from responding to stochastic fluctuations in demand by way of dynamic pricing during the selling horizon.
Alternatively, there is the option to adjust prices in a dynamic manner during the selling horizon (e.g. Lai et al. 2010, Levin et al. 2010, and Su 2010 a ). This variant is probably more intuitive and used by the vast majority of retailers who do not publicly announce their future price discounts. Here the seller does not commit to a price path at the beginning of the selling horizon, but decides on the price to charge during the selling horizon, taking into account current developments in terms of demand that are stochastic to him (for a discussion of the seller's information on demand, see Section 3.8). However, he is now restricted to subgame perfect policies, that is, loosely speaking, he cannot influence customers' decisions by pretending to stick to a certain preannounced price path; the customers will not believe an announcement that includes suboptimal prices for a future point in time, but instead assume that at this future point in time, the seller again sets prices optimally from this instant onwards. Thus, the resulting price path is such that the seller will - in equilibrium - not deviate from it in the future, even if he recalculates his prices. In this context, the well-known Coase conjecture (Coase 1972) states that with the possibility of continuous price adjustments, the only subgame perfect equilibrium price the monopolist can sustain is a uniform price equal to marginal cost. Similarly, Aviv/Pazgal (2008) point out the fact that, as part of their model, it is only typically beneficial with regard to myopic customers for sellers to keep as many options open for as long as possible and price them accordingly in a dynamic manner. In a market with strategic customers, it is better for a seller if he is able to make a credible voluntary commitment and to freely specify the prices upfront. This is an example of the game-theoretical phenomenon called "burning the bridges", meaning that limiting one's choices of future actions may put one in a better position at equilibrium. Dasu/Tong (2010) also found similar results. They show by way of numerical tests that, on average, a model with pre-announced prices is superior to a model with dynamic pricing and having two or three price changes constitutes a nearoptimal performance in terms of a theoretically possible upper limit.

### 3.3 Pricing policy

In practice, prices are adjusted only in a single direction over time in many industries. Whilst, for example, prices within passenger air travel generally increase right up to the departure date, retail prices tend to fall over time. This can be captured in models on dynamic pricing by
way of the restriction to so-called markup (prices remaining the same or increasing over time) or markdown policies (non-increasing prices). While only one paper (Biyalogorsky 2009) exclusively focuses on markup policies, about half of the research focuses on markdown policies. The majority of authors use models where markup and markdown policies are possible, which enables them to gain insights on the optimality of a markup or markdown policy. For example, Besanko/Winston (1990) allow arbitrary policies and find that only markdown policies are the optimum solution within their model. Particularly noteworthy within this context is the paper written by Su (2007), which explicitly focuses on the analysis of different policies for different buyer segments which differ in their time preference and valuation. He worked out which pricing policy is the optimum solution if a particular buyer segment represents the majority within the market. If, for example, the majority of customers are impatient and also have a high valuation, prices should decline over time; well-off customers will make their purchases early at high prices and customers with a lower valuation are also prepared to wait longer. If high-value customers are patient, prices should increase to discourage waiting. On the contrary, many of the current papers concentrate on markdown policies as a result of their focus on retailing, such as Aviv/Pazgal (2008), Gallego et al. (2008), and Lai et al. (2010).

### 3.4 Demand arrival process

The majority of relevant papers assume the arrival of customers to be simultaneous at the beginning of the selling horizon (e.g. Besanko/Winston 1990 and Levin et al. 2010). This is indeed realistic for certain applications, such as for products that are heavily advertised prior to their launch, or for innovative durable goods. In addition, it also improves the mathematical tractability of the models.
However, certain papers take into account the case that customer arrival is sequential during the selling horizon. Again, this is probably the more realistic assumption, as people become interested in many products only after their launch, for example, because they did not know about them before or did not feel the need to own them for whatever reason. Furthermore, this is also consistent with the traditional revenue management setting where airline tickets, for example, are sold in advance to customers one after another. If customers enter the market gradually, customers' arrival is typically modelled with a Poisson process, as shown in the papers by Aviv/Pazgal (2008), Elmaghraby et al. (2009), and Yin et al. (2009). The description of the arrival process in Su's work is especially diverse. In his first paper, Su (2007) assumes a constant flow of new customers into the market in accordance with the EOQ model which is well-known from production. However, in Su (2009), some customers enter the market at the beginning of the selling horizon and other customers enter later. In his most recent paper, Su (2010a) takes into account different demand segments whose customers enter the market at different times, depending on the current market situation in certain cases. Su /Zhang (2008) analyse the influence of customer arrival on their results and show that their major insights hold, regardless of whether customers arrive sequentially or simultaneously. Liu/van Ryzin (2008) state that it is more difficult to achieve price discrimination in case of a simultaneous arrival, as customers have greater flexibility in terms of their time of purchase.

### 3.5 Number of time periods

In the literature on dynamic pricing with strategic customers, the selling horizon is usually divided into several time periods. As time is regarded as discrete, there is no lapse of time in a time period, that is, the seller can set one price for each period and purchases as well as customer arrivals happen at the periods' beginning.
There are only very few papers that consider an infinite number of time periods. Su (2010b) studies a dynamic pricing problem for products with stable consumption patterns like staple
foods, which can be stockpiled by customers at some inventory holding cost. He shows that either a fixed price or periodic promotions are optimal. Specifically, periodic promotions are better when frequent shoppers have a higher valuation than occasional shoppers for the product. Following prior literature from Economics, Besanko/Winston (1990) also briefly consider an infinitely long selling horizon. In this case, revenue falls to marginal costs as the seller loses his market power because in future periods he will land in competition with himself. ${ }^{8}$
Together with about a quarter of current research papers, Besanko/Winston (1990) clearly focus on the analysis of a finite number of time periods greater than two. They point out the fact that, given that the seller's and his customers' discount rates are identical, he is in a worse position in a market with strategic customers with an increasing number of time periods, whereas he is able to profit from additional periods in a market with myopic customers. On the one hand, the reasons for this are that, if there are fewer periods, strategic customers have less time and fewer opportunities to take advantage of their strategic behaviour and exploit their power against the seller. On the other hand, if customers are myopic, then the seller does not face any sophisticated behaviour and can freely take advantage of additional time periods and the associated possibility of changing prices to skim the different levels of valuation over several periods in a more "subtle" manner. It is conspicuous that the vast majority of papers are limited to a finite number of time periods. Elmaghraby et al. (2008) point out that, under specific assumptions, an optimal markdown policy with just two or three prices exists. The paper by Bansal/Maglaras (2009) follows the same direction. Here, the seller uses the availability risk as a decision variable to discourage strategic waiting through rationing. If customers are risk-neutral, a policy with two prices is optimal; if customers have a low risk-aversion, such a policy is near optimal.

Together with its practical appeal, the above theoretical results provide a strong rationale for considering only two periods. Moreover, this significantly improves analytical tractability. About half of all papers restrict themselves to two periods and, thus, to two prices from the outset. Like Zhang/Cooper (2008) these papers mostly consider a seasonal retailer with limited capacity who seeks an optimal markdown policy with two prices. In this scenario, the price in the first period corresponds to the regular price of the product; in the second period clearance sales take place and a reduced price is offered.

Note that there are a number of papers that describe their problem setting as if it could also be considered as one in continuous time. If this does not directly translate into the model, we consider time as discrete. For example, Cachon/Swinney (2009a) and Li/Zhang (2012) explain that the customers arrive during the selling horizon; their time of arrival, relative to other customers' arrival, determines the probability that they will obtain the product in case of shortage. But in the models, these considerations are reflected in a simple parameter denoting customers' belief about their position in this waiting line. In the simplest case (see Li/Zhang 2012), customers just expect to be in the middle and this parameter equals $1 / 2$. Gallien (2006) devotes a significant part of his analysis to transforming a continuous-time problem into an equivalent discrete-time one. He considers several selling mechanisms and rationalizes the restriction to dynamic pricing by showing that, under several assumptions, a dynamic pricing mechanism, where prices increase (markup) after each sale, is optimal.

There are also some papers which explicitly consider continuous time (e.g. Aviv/Pazgal 2008, Yin et al. 2009, Osadchiy/Vulcano 2010, and Swinney 2011). These papers mostly restrict the seller to selling only at two prices while customers arrive according to a Poisson process. An exception is Swinney (2011), where all customers are present in the market upfront but may delay their purchase decision until they learn their valuation throughout the selling horizon (see also Section 4.6). Moreover, in this model, the seller also decides when to discount the
price. Osadchiy/Vulcano (2010) is the only paper where a point in time is a decision variable. Their seller decides when to discount his product.

### 3.6 Time preference

This criterion of the classification scheme captures the consideration of seller's and customers' time preferences. Time preferences express how much higher the utility of obtaining a certain surplus or revenue now is compared to the utility of a nominally equally large surplus or revenue in the future. Given that strategic customers are indeed characterised by their intertemporal utility maximization, this means that adequate consideration of the factor "time" seems particularly important. Thus, customers' as well as seller's time preferences play a critical role for the seller trying to achieve intertemporal price discrimination and trying to extract higher revenues.

A significant number of papers involve a time preference only on the part of customers. The higher a customer's time preference, the more likely it is that he will make an earlier purchase, even if he expects prices to decrease. Customers' time preference is usually motivated by the value loss caused by delayed product consumption, for example, because of loss of utility when the product cannot yet be used, or because of the reduced appeal of "not being among the first consumers" ${ }^{\prime 9}$. These behavioural effects are commonly assumed in the sale of fashion, technology, and seasonal products.

From a modelling perspective, customers' time preference is usually taken into consideration by discounting surpluses obtained in later periods through a per-period discount factor (e.g. Besanko/Winston 1990) ${ }^{10}$. This factor can either be the same for all customers or (usually stochastically) vary between individuals and can also be interpreted as customers' degree of patience (e.g. Su/Zhang 2008 and Parlatürk 2012). Other authors such as Aviv/Pazgal (2008) and Cachon/Swinney (2009a) do not discount surpluses, but only discount the valuation over time, thus resulting in minor changes in the models and results. Interestingly, there is also one single paper by Su (2009) assuming that payment can be temporally separated from consumption. This is typical for many services, such as in the airline industry, where tickets are paid for in full before air travel takes place. In that case, customers who buy early do not discount their payment but discount the valuation, whereas customers buying just before boarding discount both payment and valuation. Thus, customers aim at delaying the purchase. Note that many authors interpret the discount factor as the degree of strategic behaviour of the customer, as customers with a discount factor of zero will place no value on future surpluses (behave myopically), whereas customers with a positive discount factor between zero and one consider future surpluses, and hence may delay a purchase to the future (behave strategically).
Customers' discount factors play a significant role for the Coase conjecture (see Section 3.2), which was derived by Coase in a setting without discounting. However, if customers are less patient than the seller, that is, if customers discount, the conjecture may not apply and the seller may be able to sustain temporal price discrimination (see von der Fehr/Kühn 1995). In a problem setting with discounting customers, Cachon/Swinney (2009a) show that the seller should carefully choose the initial capacity and set prices during the selling horizon. If he decides upfront, the resulting risk of unsold capacity units at the end of the selling horizon is too great. In this context, Aviv/Pazgal (2008) show that segmenting customers according to variance in the base valuations is impossible when the customers' discount factor is close to one and, hence, customers do almost not discount. A similar result is obtained by $\mathrm{Su} / \mathrm{Zhang}$ (2008) who find that, in their setting, as the factor increases from zero to one, customers become more patient and are more willing to wait for price cuts and equilibrium prices decrease, leading to lower seller profits (Su/Zhang 2008). A similar result has been derived by Cachon/Swinney (2011).

Some authors include discounting on the part of sellers and customers in their models. Identical discount factors for sellers and customers, which are justified by assuming perfect capital markets (see Stokey 1979), are used, for example, by Besanko/Winston (1990) and Gallien (2006). Other papers (e.g. Mak 2008, Mantin/Granot 2010, and Liu/Zhang 2012) allow for different discount factors on the part of sellers and customers, whilst the customers usually have identical time preferences among themselves. For seasonal, fashionable, and durable goods, such as sporting goods, apparel, and consumer electronics, it is reasonable to assume that customers discount the value of such products faster than the firm does (meaning that the factor with which future values are discounted is smaller for customers).
In general, the results obtained in the literature when seller and customers discount with the same factor are quite similar to the results in the case that only customers discount (e.g. Besanko/Winston 1990 and Villas-Boas 2004). For example, Besanko/Winston (1990) show that there is an optimal dynamic pricing policy with declining prices. The existence of discount factors plays an important role for this outcome. In fact, in their model, if utility is not discounted over time, all customers simply purchase in the last period at the lowest price as in the Coase (1972) problem. Besanko/Winston (1990) also show that, if there is a large number of time periods for potential price changes, then an increase in the discount factor results in a lower price in each period; this leads to a profit loss. When the discount factor reaches one, price will converge to marginal cost in each period, leading to zero profit, the result that had already been emphasized by Stokey (1981). Another effect occurs if the seller can commit to a price path (see Section 3.2). Xu (2011) shows that in this case, the exact value of the discount factor is irrelevant, as all customers will purchase in the first period. This is also aligned with traditional results of the economics literature described by Stokey (1979) in the case that sellers can commit to a price path, stating that intertemporal price discrimination is not profitable when the seller and customers share the same discount factor; the seller should just charge the static monopoly price at the very beginning.
Regarding different discount factors of customers and seller, Mak (2008) shows in his setting that if the seller does not discount future profits while customers have time preference, it is always optimal to use a price skimming strategy even in the presence of strategic waiting. However, if the seller discounts future profits with a discount factor being higher or equal to the customers' one, it is not optimal to attempt price skimming when inventory is not sufficiently high; the seller should instead sell his inventory at the beginning of the season. A similar result is obtained by Osadchiy/Vulcano (2010) who find that the most beneficial scenarios occur when the seller's discount factor is lower than the customers' and, thus, the seller can take advantage of the customers' impatience.
In the majority of papers (e.g. Elmaghraby et al. 2009, Yin et al. 2009, and Dasu/Tong 2010) the time preference of sellers or customers is not considered at all. ${ }^{11}$ These papers usually explicitly justify the reasons why the discount factor is not included in their model. For example, Dasu/Tong (2010) state that a discount factor is not required as they are only considering a very short selling horizon. Liu/van Ryzin (2008) argue that "they ignore discounting partly as a practical approximation, but more importantly to isolate and study the effect that rationing risk alone has on customer behaviour". Some papers also show as an extension that the results obtained in their specific settings without discounting directly transfer to settings with discounting (e.g. Elmaghraby et. al. 2008 and Liu/van Ryzin 2008).

### 3.7 Market type

Almost all papers consider markets in their models in which there is a single seller, that is, a monopoly (e.g. Cachon/Swinney 2009a and Mersereau/Zhang 2012). By avoiding competition between multiple sellers, the authors significantly reduce the mathematical complexity of
their models, which already contain the interaction between customers and sellers and between customers themselves. While a monopoly is usually not mirroring reality, it can be argued that the resulting modelling error is not too big for many practical applications; the influence of competition is captured at least to some extent in the historical data used, for example, to calibrate the demand forecast (e.g. Phillips 2005, p. 55).
However, an oligopoly generally is a more realistic depiction of reality in many sectors and markets. Liu/van Ryzin (2008) consider an oligopoly and point out that competition reduces the market power and therefore the profits of each individual seller. The availability risk, which typically has an impact on customers' purchasing decisions, is significantly weakened as it is possible for customers to swap to a different seller without any restrictions. In addition, a critical number of sellers exist, above which it is no longer appropriate for a seller to subject customers to a capacity shortage. Starting from their earlier (but published later) paper on a monopolist (Levin et al. 2010), Levin et al. (2009) examine equilibria for different levels of competition; i.e. monopolies, duopolies, and oligopolies. Another paper was written by Liu/Zhang (2012), who extend the traditional model from Besanko/Winston (1990) (see Section 3.1) for oligopolies and whose sellers manage products of differing levels of quality. Aviv/Pazgal (2008), Elmaghraby et al. (2009), and Lai et al. (2010) examine oligopolies with respect to their proposals for potential model extensions. A special case is Mak et al. (2012), where a customer alternately considers two sellers.

### 3.8 Information Setting

Both the seller's decision on his pricing as well as buyers' decisions on the timing of their purchases depend heavily on the information individually available to these players. The seller basically makes use of this information to set prices optimally. Customers use it mainly to estimate future prices if those are not announced upfront (see Section 3.2) and to assess the probability of being able to obtain the product in the future. In this context, see also the problem extension "additional purchase options" (especially reservations, Section 4.3). Given the importance and complexity of information availability, this subsection differs from the previous ones insofar as we develop a tabular classification that exclusively focuses on information availability. For each paper, we describe the information setting considered in detail with regard to seven exogenous aspects; they are clustered into the four groups "capacity", "market size/customer arrival", "valuations", and "time preferences" (see Table 3). In line with the literature on game theory (e.g. Fudenberg/Tirole 1991 and Myerson 1997), we distinguish complete and incomplete information with regard to each of the seven aspects. Generally, a player has complete information on an aspect if he knows everything necessary to make his decisions. If he has incomplete information, he lacks some information that would be relevant for his decision. In the penultimate column of Table 3, the information setting is summarized; this corresponds to the element "information setting" in the general classification (see Tables 1 and 2). Similarly to Table 2 , we use brackets if a paper additionally considers an information setting which is given considerably less attention than the focused one. If a certain aspect is not applicable to a paper, this is indicated by "--". ${ }^{12}$
The first group contains only one aspect (first aspect in Table 3) and relates to customers' information on capacity. In reality, there are a couple of companies that inform customers about their inventory level in different ways. Some companies permanently make known their currently available capacity. Examples include web-based travel agencies such as Expedia.com, where customers are able to view the availability of airline tickets on specific flights before they decide to purchase, Filene's Basement (see Section 3.2), and Benetton. Online retailer Amazon displays inventory levels when stock is low and sierratradingpost.com, another online retailer, uses the tag "almost gone!" next to the products to indicate that the sell-
out risk is currently very high. Other companies only reveal to customers whether at least one unit is still available. All these ways of disclosing inventory are easy to implement, particularly with respect to selling over the Internet. In general, inventories are often disclosed for products with a short selling horizon, notably seasonal products, for which inventory is often non-replenishable over the selling horizon similar to the fashion products referred to in Section 4.1 (e.g. Mak 2008).
In most of the models presented in the literature, customers have complete information about the seller's exogenous initial capacity. Incomplete information about capacity is considered by Mak (2008), Dasu/Tong (2010), and Yousefi et al. (2011). The latter consider two scenarios where either demand or capacity is uncertain and, thus, customers are uncertain about product availability. The first two papers directly compare scenarios where customers have complete and incomplete information about capacity. Dasu/Tong (2010) show that the seller benefits from not disclosing his inventory. Mak (2008) obtains completely different results. In his two-period model, the customers try to infer the seller's privately known inventory as well as the second period price from the first period price. He shows that the seller cannot be in a better position by concealing his capacity. He attributes this to the fact that, in his model, the seller cannot commit to a price path at the beginning of the selling horizon, but decides on the price to charge during the selling horizon.
Moreover, Mak (2008) and Yin et al. (2009) analyse whether to disclose inventory if it is a decision variable of the seller (see also Section 3.1). Here, customers are not able to infer the capacity decision because no rational expectations equilibrium (see Endnote 12) is used or because they can only infer some inventory range. In this context, Mak (2008) shows that a seller deciding on inventory should still disclose his capacity. By contrast, Yin et al. (2009) assume that the seller can commit to a price path in advance and find that higher revenues can be achieved if only availability (that is, whether at least one unit is still in stock) is published. This is because the greater level of uncertainty compels customers to purchase earlier and to pay a higher price on average. When the seller moves from displaying his inventory to not displaying it, the majority of the benefit can be obtained without adapting the capacity decision. In general, it has to be ensured that the disclosed capacity is highly credible. In a recent paper related to this topic, Allon/Bassamboo (2011) ${ }^{13}$ point out that disclosed inventories are worthless for rational customers in cases where the published values are doubtful and cannot be verified.

The second group relates to knowledge about market size and customers' arrival over time for both the seller and the customers (second and third aspect in Table 3). Market size is sometimes also called total or potential customer population and is relevant no matter whether customers are present in the market from the beginning or arrive during the sales period (see Section 3.4), whereas, obviously, arrival time is only relevant when customers arrive sequentially over time. Knowledge about time of arrival relates only to other customers, as customers usually have private information about their own time of arrival. If multiple customer segments are considered, in our classification, "market size" relates to "size of each segment". Interestingly, our analysis shows that in almost all papers the customers' and the seller's information is the same in this aspect. ${ }^{14}$
About half of all papers consider incomplete information on the market size and customers' arrival time. If customers arrive simultaneously upfront, the market size is usually stochastic and follows an arbitrary distribution, which is either continuous (e.g. Lai et al. 2010) or discrete (e.g. Huang/van Mieghem 2011). Many papers make further assumptions to derive their results. For example, Wang/Hu (2011) assume a uniform distribution in part of their analysis. If customers arrive over time and time is modelled discrete, the above applies to every period where customers can arrive (e.g. Kim/Swinney 2011). If time is continuous, customer arrival
usually follows a homogeneous (e.g. Aviv/Pazgal 2008) or nonhomogeneous Poisson process (e.g. Osadchiy/Vulcano 2010).

Other authors consider participants with complete information on the number of customers and their arrival. In this case, again, customers are discrete or continuous, that is, infinitesimally small. If they are considered discrete and enter the market upfront, the number of customers or the number of customers in each segment is commonly known (e.g. Elmaghraby et al. 2008). If they are considered continuous, total market size is often normalized to one (e.g. Besanko/Winston 1990). If discrete customers arrive over time, the number of customers entering the market at each point in time is given (e.g. Yu et al. 2011 and Bhalla 2012). In addition, customers can also arrive continuously at some rate in continuous time (e.g. Su 2007).

There are only a few papers that consider both complete and incomplete information on this aspect and compare the results. Gallego et al. (2008) show that, in their setting, when four conditions - among others, demand being known - are satisfied, a single static price is optimal, whereas, when the seller is uncertain about total demand or its composition (that is, valuations; see below), a markdown policy may become optimal. Jerath et al. (2010) compare a dynamic pricing policy with a clearance sale at the end of the selling horizon ("last-minute selling") to a policy that seeks to segment the market with an intermediary's help that offers a so-called opaque product (e.g. Gönsch/Steinhardt 2012) and find for a setting with incomplete information that as the probability of demand exceeding capacity increases, firms start to prefer opaque sales over last-minute sales. However, the authors show that if information on demand is complete, last-minute sales are never offered, but opaque sales are used if customer valuations are very low. In a quite unusual setting with availability guarantee, Borgs et al. (2011) show that their results do not depend on having complete or incomplete information about capacity and customer arrival.

The third group relates to seller's and customers' information on valuations (fourth and fifth aspect in Table 3). It is a common assumption that customers always know their own individual valuations. ${ }^{15}$ Thus, in the fourth aspect, we indicate only whether they know the valuations of other customers. To the seller, individual valuations are generally unknown. Having this information, obviously, the seller would not use dynamic pricing but instead would try to charge exactly these valuations and thereby completely skim customer surplus. ${ }^{16}$ Overall, we categorize seller's information about valuations as complete if the seller has perfect information about customers as a whole. ${ }^{17}$

The majority of research analyses markets where valuations are known, that is, either customers are considered as continuous and the density function of the valuations in the market is known (e.g. Besanko/Winston 1990) or only very few discrete levels of valuations are possible and these values are known (e.g. Su/Zhang 2008, Cachon/Swinney 2011, and Tereyagoglu/Veeraraghavan 2012). In the latter case, these segments are often described as low and high type customers (e.g. Li/Zhang 2012).
About a quarter of the literature focuses on problems where information on valuations is incomplete. For example, in Osadchiy/Vulcano (2010) only the time dependent distribution of (discrete customers') valuations is known. In Su (2009), the valuations of the high and the low customer segment are known, but total market size and each customer's individual segment is stochastic. Note that seller's and customers' information on valuation is also incomplete in the literature discussed in Section 4.6, where customers do not even privately know their valuations.

A number of papers consider both complete and incomplete information on this aspect and compare the results. Dasu/Tong (2010) briefly consider public valuations (and, in their model, hence deterministic demand) and show that under these conditions, a simple two-price policy
emerges. Thus, they turn to stochastic demand throughout their paper, where more price changes are needed. Mersereau/Zhang (2012) develop a robust policy for a setting where valuations are known but the fraction of strategic customers is unknown. They show that this policy still works well if information is incomplete because the number of discrete customers with valuations above a certain value is known only up to its Poisson distribution. As already mentioned above, Gallego et al. (2008) show that when valuations become uncertain, the seller should move from a single-price policy to a two-price markdown policy. Elmaghraby et al. (2008) use a model with an arbitrary but finite number of time periods (and, thus, number of price changes) to show that under complete information regarding valuations, the optimal markdown policy has two price steps; buyers with multi-unit demand either do not buy or buy up to their demand (all-or-nothing; see also Section 4.7) at each price step. When they relax the strict assumption that the set of customer valuations is known to the seller, customer valuations are drawn from known distributions. This leads to a stochastic element and a degree of uncertainty in the equilibrium calculations. Now, the optimal markdown policy has at most three steps and customers still buy all-or-nothing at each price step.
The last group relates to the time preferences (sixth and seventh aspect in Table 3). Customers usually know their individual time preference and have the same knowledge as the seller regarding other customers' time preference. The seller is considered to have complete information if he knows customers' time preferences exactly, which is by far the predominant case and usually coincides with time preferences being homogeneous (e.g. Liu/Zhang 2012). The seller's information is incomplete if time preferences are heterogeneous (Cachon/Swinney 2009a). In all papers, customers' knowledge about the seller's time preference is complete. Note that we consider this aspect non-applicable if prices are announced upfront, because customers do not need to bother about the seller's time preference in this case (e.g. Osadchiy/Vulcano 2010). Unfortunately, to the best of our knowledge, no paper elaborates on the influence of information about time preferences. The only study that can be considered as involving both complete and incomplete information on valuations is from Gallego et al. (2008), who in fact draw time-dependent valuations from a distribution. Thus, their model can clearly be seen to contain a privately known individual discount factor, but the effects of unknown valuations and discount factor cannot be isolated.

Finally, the overall information setting (last column in Table 3) corresponds to the element in the general classification (Table 1). Here, the information setting is described as incomplete if some information is not completely known to the seller or the customers and the resulting model is stochastic. The majority of research follows this approach. On the other hand, if everyone has full information on all aspects, the information setting is complete. Only a quarter of research exclusively considers complete information settings (e.g. Besanko/Winston 1990), usually to deliberately remove uncertainty from the model, as Borgs et al. (2011) point out. These papers use deterministic models, where consideration of deterministic demand must be justified in particular. The exact knowledge of discrete demand is often justified by the large market assumption (e.g. Bansal/Maglaras 2009). Another possibility is the use of so-called fluid models, where demand (that is, the number of customers with valuations above a certain threshold) is replaced by its expected value (e.g. Gallego et al. 2008 or Yu et al. 2011).

Table 3: Information setting (CuIn: Customers' information, SeIn: Seller's information, CI: Complete information, II: Incomplete information)

| Reference | $\begin{gathered} \hline \text { CuIn } \\ \text { on } \\ \hline \end{gathered}$ | SeIn <br> on | $\begin{gathered} \hline \text { CuIn } \\ \text { on } \\ \hline \end{gathered}$ | SeIn on | $\begin{gathered} \text { CuIn } \\ \text { on } \\ \hline \end{gathered}$ | $\begin{array}{cc} \hline \text { SeIn } & \text { CuIn } \\ \text { on } \mathrm{Cu} & \mathrm{Se} \\ \hline \end{array}$ | Overall |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Capa- } \\ \text { city } \end{gathered}$ | $\begin{array}{r} M c \\ \text { Size/C } \\ A r \end{array}$ | stomer <br> al | Valua | ations | Time Prefer ences | Information Setting | Comment |


| Aviv/Pazgal (2008) | CI | II | II | II | II | CI | -- | II |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bansal/Maglaras (2009) | -- * | CI * | -- * | CI | -- * | -- | -- | CI | * Availability risk ("fill rate") known to customers |
| Besanko/Winston (1990) | -- | CI | CI | CI | CI | CI | CI | CI |  |
| Bhalla (2012) | -- | CI | CI | II * | II * | CI | CI | II | * Participants are unaware of product's value, but customers receive a private signal |
| Biyalogorsky (2009) | CI | II | II | CI | CI | -- | -- | II |  |
| Borgs et al. (2011) | -- * | CI, II | -- * | CI | CI | -- | -- | CI, II | * Seller guarantees service to every customer, despite constrained capacity |
| $\begin{aligned} & \text { Cachon/Swinney } \\ & \text { (2009a) } \end{aligned}$ | -- | II | II | CI | CI | II | -- | II |  |
| Cachon/Swinney $(2011)$ | -- | II | II | CI | CI | CI | -- | II |  |
| Chen/Zhang (2009) | -- | CI | CI | CI | CI | CI | CI | CI |  |
| Dasu/Tong (2010) | CI, (II) | CI, (II) | CI, (II) | II, (CI) | II, (CI) | -- | -- | II, (CI) |  |
| Elmaghraby et al. (2008) | CI | CI | CI | CI, II | CI, II | --, (CI) | -- | CI, II |  |
| Elmaghraby et al. (2009) | CI | II, (CI) | II, (CI) | CI, II | CI, II | -- | -- | II, (CI) |  |
| Gallego et al. (2008) | -- * | CI, (II) | -- * | CI, (II) | -- * | CI, (II) | -- | CI, (II) | * Customers directly consider availability risk in second period |
| Gallien (2006) | CI | II | II | II | II | CI | CI | II |  |
| Gallien/Gupta (2007) | CI | II | II | II | II | CI | -- | II |  |
| Huang/van Mieghem (2011) | -- | II | II | CI | CI | CI | -- | II |  |
| Jerath et al. (2010) | CI | CI, II | CI, II | CI | CI | -- | -- | CI, II |  |
| Kim/Swinney (2011) | -- | II | II | CI | CI | -- | -- | II, CI |  |
| Koenigsberg et al. (2008) | -- * | CI, II | CI, II | CI | CI | -- | -- | II | * Customers directly consider probability of last-minute discounts |
| Lai et al. (2010) | -- | II | II | CI | CI | CI | -- | II |  |
| Levin et al. (2009) | CI | CI | CI | II | II | CI | -- | II |  |
| Levin et al. (2010) | CI, (--) | CI | CI | II | II | CI | -- | II |  |
| Li/Zhang (2012) | -- | II | II | CI | CI | -- | -- | II |  |
| Liu/Xiao (2008) | CI, (--) | II | II | II | II | -- | -- | II |  |
| Liu/van Ryzin (2008) | -- | CI | CI | CI | CI | --, (CI) | -- | CI |  |
| Liu/Zhang (2012) | -- | CI | CI | CI | CI | CI | CI | CI |  |
| Mak (2008) | CI, II | CI | CI | CI | CI | CI | CI | CI, II |  |
| Mak et al. (2012) | -- | CI | CI | II | CI | CI | -- | II |  |
| $\begin{aligned} & \text { Mantin/Granot } \\ & \text { (2010) } \end{aligned}$ | -- | CI | CI | CI | CI | CI | CI | CI |  |
| Mersereau/Zhang (2012) | CI | $\begin{aligned} & \mathrm{CI} *, \\ & \mathrm{II} * \end{aligned}$ | $\begin{aligned} & \mathrm{CI} *, \\ & \mathrm{II} * \end{aligned}$ | $\begin{aligned} & \mathrm{CI} * \\ & \mathrm{II} * \end{aligned}$ | $\begin{aligned} & \mathrm{CI} *, \\ & \mathrm{II} * \end{aligned}$ | -- | -- | $\begin{aligned} & \mathrm{CI} \text { *, } \\ & \text { II * } \end{aligned}$ | * Proportion of strategic customers not known <br> \# Number of customers with valuation above a certain level stochastic |
| Nair (2007) | -- | CI | CI | II, (CI) | II, (CI) | CI | CI | II, (CI) |  |
| Osadchiy/Vulcano (2010) | -- | II | II | II | II | CI | -- | II |  |
| Parlatürk (2012) | --, (CI) | CI | CI | CI | CI | CI | -- | CI |  |
| Prasad et al. (2011) | -- | II | II | II | II | -- | -- | II |  |
| Su (2007) | $\begin{gathered} \hline \text { CI *, } \\ (--) \end{gathered}$ | CI | CI | CI | CI | CI | -- | CI | * Customers directly consider availability announced |
| Su (2009) | CI | II | II | CI | CI | CI | -- | II |  |
| Su (2010a) | CI, (--) | II | II | CI | CI | -- | -- | II |  |
| Su (2010b) | -- | II * | II * | CI | CI | -- | -- | II | * Demand depends on privately known customers' inventory |
| Su/Zhang (2008) | -- | II | II | CI | CI | --, (CI) | -- | II |  |
| Swinney (2011) | -- | II | II | CI | CI | CI | -- | II |  |


| Tereyagoglu/ <br> Veeraraghavan <br> (2012) <br> Villas-Boas (2004) | -- | II | II | CI | CI | -- | -- | II |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Wang/Hu (2011) | -- | II | CI | CI | CI | CI | CI | CI |  |
| Xu (2011) | -- | CI | CI | CI | CI | II $*$ | CI | CI | II | * Stochastic drop of valuation

### 3.9 Common combinations

After having explained the classification criteria in isolation in the previous subsections, we now discuss common combinations of criteria's values that are looked at in the literature. First of all, note that Table 2 in Section 2 impressively reflects the already discussed heterogeneity in the literature: There are only very few papers that exactly consider the same setting in terms of the same value for all criteria, especially when the problem extensions are also considered. However, there are some quite typical industry application domains that are often discussed in the context of dynamic pricing with strategic customers and that appear with common combinations for at least some of the classification criteria. These application domains are online services (e.g. cloud computing), the airline industry, and seasonal fashion retailing.
With respect to these typical domains, the classification criteria can be divided into three groups:

The first group comprises the criteria "capacity" and "number of time periods". Regarding these criteria, basically the same assumptions hold for all three domains. Online services, airline industry as well as seasonal fashion retailing all have sparse and/or expensive capacity and thus capacity is usually modelled as "limited" or as decision variable. Furthermore, the number of time periods is limited ("two" or "finite"), because capacity is considered perishable and leftover inventory has no worth after each online service fulfilment period, take-off, or the end of the selling horizon, respectively.
The second group comprises the criteria "demand arrival process", "time preference", "information setting", and "market type". In these criteria, there is no specific relation prevalent in the literature with regard to the three application domains. Although it is quite clear that specific assumptions should hold good for these criteria, there are models for the abovementioned industries with different combination of values for all three criteria. This is mainly because these criteria are often defined by the authors in order to end up with mathematically tractable models rather than to model the real-world setting appropriately. For example, it is quite obvious that airlines or retailers are faced with an oligopolistic setting in reality. However, in order to end up with manageable models, the vast majority of authors use a monopoly setting as simplification, arguing that the loss due to the lack of modelling accuracy is limited (also see the discussion in Section 3.7).
The third group comprises the two remaining criteria "time of pricing" and "pricing policy". The assumptions regarding these criteria are different for the three application domains under consideration and are discussed in more detail in the following paragraphs. In this context, we also refer to the relevant papers for each of the application domains.

In the market for online business services, such as cloud computing, a firm sells computation-on-demand to its customers. Most of the customers typically ask for reliable services and do not accept any rationing. It is seen to be the firm's responsibility to set prices which ensure that all service requests can be fulfilled with the limited service capacity. Therefore, with respect to the criterion "pricing policy", markups as well as markdowns are permitted. The models of Koenigsberg et al. (2008), Su (2009), Levin et al. (2010), and Borgs et al. (2011) are in line with these requirements. Although the model of Koenigsberg et al. (2008) was originally designed for the low-cost carrier easyJet, it could be transferred to a cloud computing pricing problem. Within the criterion "time of pricing", most of the authors assume that prices can be adjusted during the selling horizon. However, Borgs et al. (2011) propose a model where prices are announced upfront at the beginning of the selling horizon, probably to increase analytical practicability.

With regard to the airline industry, three different subdomains are usually distinguished: In the case of the conventional network carriers, regarding pricing policy, it is convenient to assume that only markup pricing is possible. This is justified because leisure travellers, who usually have a high valuation, tend to appear early in the selling horizon, while business travellers with a higher valuation tend to appear late (the well-known "low before high"assumption in revenue management). With respect to the time of pricing, prices can be decided throughout the selling horizon, even though there are a few authors who assume that prices are announced at the beginning of the selling horizon for ease of modelling (see, e.g. Biyalogorsky 2009). The second subdomain comprises the low-cost carriers. Here, usually markdowns as well as markups are allowed, and prices are decided throughout the selling horizon (see Koenigsberg et al. 2008). The third subdomain comprises charter airlines offering last-minute discounts. Here, the pricing policy is assumed to be purely markdown, and the time of pricing is during the selling horizon. Jerath et al. (2010) and Lai et al. (2010) present corresponding models. They incorporate only two periods, and the discount price is not announced until the beginning of the second period. This type of model is very popular in the travel industry in general (e.g. airlines, hotels, tour operators, and car rental companies).
Seasonal fashion retailing is one of the most popular fields of application in the literature on dynamic pricing with strategic customers. Regarding the pricing policy, only markdowns are considered (e.g. Aviv/Pazgal 2008, Elmaghraby et al. 2008, Gallego et al. 2008, and Dasu/Tong 2010). In general, customers with a high valuation, who want to be among the first to wear a new collection and choose from the full variety of different products, visit the shop early in the selling horizon. Customers with a lower valuation wait for discounts during the selling horizon and visit the shop at a later time. Although it is not as obvious as in the airline industry that leftover inventory has no worth, also in the fashion industry clothes that are "out of date" after the end of the selling horizon can be assumed to be worthless. Regarding the time of pricing, it can be assumed that the price decision is made during the selling horizon (Aviv/Pazgal 2008 and Dasu/Tong 2010). However, there are again authors who exclusively assume that prices are announced at the beginning of the selling horizon for ease of modelling (see Elmaghraby et al. 2008 and Gallego et al. 2008). One special case of the markdown pricing policy is examined by Elmaghraby et al. (2009) and Yin et al. (2009). They model only two periods and prices which are explicitly announced at the beginning of the selling horizon. The associated pricing policy is called "pre-announced clearance price policy" and has been adopted by many companies including J.C. Penny and Dress For Less (see Elmaghraby et al. 2009).

## 4 Extensions

In this section, we discuss the different problem extensions that are included for each paper in the last column of Table 2. As stated before, these extensions are primarily motivated by specific practical applications that are investigated by the corresponding papers' authors, depicting certain excerpts of reality in greater detail by including additional aspects in the models. Table 4 provides an overview of the different extensions found in the literature. For each paper, we state the corresponding practical setting or industry to which the authors refer in order to motivate their extension. ${ }^{18}$ In cases where relevant firms are explicitly mentioned in a paper, they are listed in brackets right after the industry.
In the following subsections, we discuss each of the extensions in detail. Therefore, for each extension, we first informally describe the idea of the extension and refer to the corresponding practical setting and real-world examples. Subsequently, we refer to the relevant publications from the literature and state the major findings.

Table 4: Overview of problem extensions

### 4.1 Short-term replenishment

- Cachon/Swinney (2009a): fashion retailer (Zara)
- Cachon/Swinney (2011): fashion retailer (Zara, H\&M, Benetton)
- Swinney (2011): automotive industry (General Motors)
4.2 Price matching
- Lai et al. (2010): office products (Staples), consumer electronics (Best Buy, Circuit City), apparel retailer (Gap), travel industry (Priceline)
- Li/Zhang (2012): --
- Xu (2011): fashion retailer (J. Crew), electronic retailer (Best Buy), car manufacturer, department stores
4.3 Additional purchase options
- Biyalogorsky (2009): airline industry
- Elmaghraby et al. (2009): department stores (Nordstrom)
- Li/Zhang (2012): telecommunication corporations (Apple, Nokia), consumer electronics (Nintendo, Playstation)
- Osadchiy/Vulcano (2010): retail warehouses (Sam's Club)
- Prasad et al. (2011): manufacturing and retailing sector, magazine and newspaper retailer, online retailer (Amazon)
- Yu et al. (2011): airline industry, travel industry, retail sector - especially toys, books and consumer electronics


### 4.4 Speculators

- Su (2010a): online marketplaces (eBay, StubHub, TicketsNow) - especially reselling sports tickets and consumer electronics


### 4.5 Auctions and mechanism design

- Bansal/Maglaras (2009): --
- Elmaghraby et al. (2009): --
- Gallien (2006): online auctions (eBay)
- Gallien/Gupta (2007): online retailer (Amazon), online auctions (eBay)
- Yousefi et al. (2011): electronic commerce (Pricetack)


### 4.6 Uncertainty in terms of customers' individual valuation

- Bhalla (2012): durable goods
- Koenigsberg et al. (2008): airline industry - especially low-cost carrier (easyJet, Ryanair, Southwest, JetBlue)
- Levin et al. (2009): airline industry
- Levin et al. (2010): travel industry - especially hotels, sports and entertainment industry
- Liu/Xiao (2008): airline industry, hotel industry, fashion retailing
- Prasad et al. (2011): fireworks, Christmas toys
- Su (2009): airline industry
- Swinney (2011): telecommunication corporations (Apple), consumer electronics (Nintendo), automotive industry, media - especially books, movies, music, video games
- Xu (2011): fashion retailer
- Yu et al. (2011): sports, concerts, airline industry, service industry, fashion retailing


### 4.7 Demand for multiple product units

- Elmaghraby et al. (2008): B2B markets
- Su (2010b): consumer goods
4.8 Customers' and seller's risk preference
- Bansal/Maglaras (2009): --
- Liu/van Ryzin (2008): consumer electronics, household appliances, high-end stores
- Prasad et al. (2011): --
- Wang/Hu (2011): --
- Yousefi et al. (2011): --
- Yu et al. (2011): --
4.9 Product quality choice
- Kim/Swinney (2011): consumer electronics, fashion retailer, automotive industry
- Parlatürk (2012): books (F+W Media, Macmillan Publishers), consumer electronics (Apple)


### 4.1 Short-term replenishment

Most firms in the seasonal fashion industry face the situation that production and ordering cycles are much longer than the selling horizon, so that replenishment is not possible. However, there are some firms that organize their supply processes so that lead times are much shorter. Spanish fashion producer and retail chain Zara is one of the most popular examples in the literature of a firm which intentionally applies such a strategy enabling short-term replenishment. Zara limits the inventory in its retail stores to create a sense of urgency among customers and to make them purchase early in the selling horizon. Thereby, it also minimizes the number and severity of markdowns in its stores in order to drive up profits. To execute this strategy, Zara monitors and replenishes the store's inventory quite frequently (see Cachon/Swinney 2009a and 2011).
Cachon/Swinney (2009a) include short-term replenishment (also called "quick response") in their model, allowing for additional flexibility for monopolists. They can make use of this flexibility in the short-term in order to respond to any developments in terms of demand during the selling horizon. The seller disposes of a specific initial capacity available at the beginning of the selling horizon which caused him relatively low unit costs. As soon as the seller has improved certainty of demand during the selling horizon, then he may increase his stock at increased unit costs. The authors show in their real-world example from the clothing retail industry that the profit surplus made as a result of restocking in the model is $67 \%$ higher on average if there are only strategic customers present within the market, as opposed to when myopic customers are present. They conclude that the possibility of performing short-term replenishments is especially beneficial when strategic customer behaviour can be assumed. In a subsequent paper, Swinney (2011) derives - within a somewhat modified model framework which is based on goods that are more complex and difficult for customers to value (see Section 4.6) - a number of conditions under which restocking in the presence of strategic cus-
tomers increases or reduces the profits of the seller. In a recent paper, Cachon/Swinney (2011) assume that all customers have an identical valuation and also investigate the possibility of increasing their valuation by way of product improvements.

### 4.2 Price matching

There are a number of real-world examples where sellers ensure that their customers are able to fully benefit from any price reductions that eventually follow their purchase. In such cases, customers are refunded the difference between their purchase price and the subsequent price. This ensures that customers never find themselves in a worse position by purchasing early and thus eliminates any benefit of delaying a potential purchase. Such so-called "price matching policies" can be found in various companies from different industries, for example, for office products (Staples), for electronic products (Best Buy), for fashion (apparel) products (Gap), and for travel tickets (Priceline).

Lai et al. (2010) compare such a price matching approach with a traditional dynamic pricing policy and show that, although price cuts can be associated with significantly high costs for sellers when they are committed to refund the price difference to existing customers, the seller's profits tend to increase whereas the consumer surplus decreases. This is achieved by a decrease in the strategical waiting time of customers, which incurs significant revenue losses in the traditional setting. However, under certain conditions, even a Pareto improvement can be achieved for all market participants. In addition, the authors show that the commitment to a price path at the beginning of the selling horizon rarely provides any benefit to sellers who are able to offer refunds as described. Li/Zhang (2012) present another paper which considers the option of refunds in the event of price cuts. Interestingly, the authors show that price guarantees do not necessarily help to mitigate the negative impact of pre-orders regarding the seller's ability to perform temporal price discrimination (see Section 4.3). In Xu (2011), the seller decides upfront how long after his purchase a customer can make use of price matching and what portion of the price difference is refunded. When customers' valuations decline over time, a finite-length price matching policy allows the seller to commit to not lowering his price too soon. Thus, he does not suffer from the Coase conjecture (see Section 3.2) and is able to do some intertemporal price discrimination.

### 4.3 Additional purchase options

In addition to the immediate purchase of a product, some sellers offer other options to customers, e.g. different implementations of advanced selling and reservations. Advanced selling (pre-orders), for example, is the sale of a product to customers in advance of delivery. It is often implemented in the manufacturing and retailing sectors by firms facing newsvendor problems. This problem entails the retailer needing to purchase his inventory before the start of a short selling horizon with uncertain demand. In such cases, advance selling can reduce the seller's inventory risk, because it decreases demand uncertainty upfront by committing customers to advance orders. Besides, booksellers such as Amazon, consumer electronics producers such as Apple, Nokia, Nintendo, and Sony often offer advance purchase discounts for soon-to-be released items (Prasad et al. 2011 and Li/Zhang 2012). Another strategy is to offer customers the possibility of making a reservation by subscribing to a kind of waiting list for end-of-season clearance. If a customer reserves the product and there is still capacity available at the end of the selling horizon, then the customer finally has to purchase it at the reduced price he already knew and committed to in advance. This strategy is implemented, for example, by entrepreneurial retailers such as Nordstrom. Sam's Club also used for a few years in the early 2000s a so-called "plunging price" to clear excess inventory (Osadchiy/Vulcano 2010).

Prasad et al. (2011) and Li/Zhang (2012) consider advanced selling options and present models that include capacity decisions. They show that by means of pre-orders transacted prior to the initial capacity decision of the seller, customers are able to exclude themselves from the availability risk. Yu et al. (2011) analyse how the optimal strategy for advance selling rests on the interdependence of customer valuations, as well as other parameters such as unit cost, capacity level, advance and spot market size. They show that a change in valuation interdependence can lead to significantly different policies for the seller. For example, when there are highly diverse individual valuations and a large customer population, the seller must offer a discount during advance selling, but he may limit the advance sales. On the other hand, when valuations are highly correlated, the seller can charge a higher price during advance selling. Moreover, for the same valuation interdependence, the qualitative nature of the optimal strategy changes with available capacity.

Elmaghraby et al. (2009) consider reservations for the end-of-season sale as described above. The authors show that, despite the additional purchase option, customers find themselves in a worse position and providers are in a better position in the Nash equilibrium than in a model without this option. That is, additional purchase options do not necessarily improve customers' positions but increase revenues and mitigate strategic customer behaviour. Biyalogorsky (2009) as well as Osadchiy/Vulcano (2010) examine the advantages of reservations in similar settings. Biyalogorsky (2009) shows that through reservations, the provider can enforce customers to request the product in an order corresponding to their valuation ("low before high"). This creates the basis for temporal price discrimination. By contrast Li/Zhang (2012) show in their setting that there is a potential conflict between advance demand information the seller obtains from offering pre-orders and his ability to perform temporal price discrimination. Accurate demand information may increase product availability in the regular selling period; this prevents the seller from charging a high price to the high-value customers in the pre-order season.

### 4.4 Speculators

In many real-word applications, besides sellers and customers, a third group of market participants plays an important role, the so-called speculators. Speculators purchase the product with the sole motivation of selling it at a profit at a later date, and are therefore in competition with conventional sellers. The Internet resale industry has steadily risen and become increasingly acceptable with the growth of companies such as StubHub (a subsidiary of eBay) and TicketsNow (owned by Ticketmaster). The total revenue from online ticket sales is estimated to reach $\$ 4.5$ billion by 2012, which is a significant increment over the $\$ 22$ billion in "regular" sales by U.S. live music and sporting event industries (see Mulpuru/Hult 2008).

Su (2010a) takes into account speculators in a dynamic pricing model with strategic customers. He shows that, despite the resulting competitive situation, speculators actually increase the profits of the seller. This is because they assume part of the risk. As a result, in cases where there is low demand, unsold products remain not only in the seller's store, but also the speculator's store. In cases where there is high demand, speculators can help to skim a large proportion of the consumer surplus within the market. In addition, customers tend to hold speculators rather than sellers responsible for any shortages and increased prices.

### 4.5 Auctions and mechanism design

Especially in the Internet business, many new selling mechanisms have evolved over the last couple of years. Online marketplaces are of growing importance. Besides platforms offering traditional online auctions like eBay, there are a number of other service providers offering innovative mechanisms in order to sell their products. A well-known example is Priceline,
offering the so called "Name-Your-Own-Price" mechanism, which makes it possible for customers to submit bids via the Internet for products such as flights or hotel stays. The concept of the "Name-Your-Own-Price" mechanism is illustrated by Anderson/Wilson (2011) in a survey specially dedicated to this subject. An introduction and further references are also included in Spann et al. (2004) and Hinz et al. (2011).
Some authors jointly consider classical auctions and dynamic pricing. For example, Gallien/Gupta (2007) explore a setting where an item is offered for sale through an online auction and a permanent buyout option remains available until it is exercised or until the auction ends. Such a buyout price may be static or dynamic and vary as the auction progresses. Through numerical experiments they find that when the seller or customers have time preferences, the seller significantly increases his utility through a buyout option, but dynamic buyout prices do not provide a substantial advantage over static ones. Elmaghraby et al. (2009) briefly examine using a second price auction instead of a markdown to dispose of unsold inventory.

Regarding mechanism design, different relationships to dynamic pricing are present in the literature. Bansal/Maglaras (2009) use techniques from that area to analyse the dynamic pricing problem of a monopolist facing strategic customers who differ in their valuations and risk preferences. To do so, the problem is reformulated as a static mechanism design problem. Gallien (2006) starts his analysis from a mechanism design point of view. He does not restrict himself to a certain sales format like dynamic pricing, but allows for arbitrary sales mechanisms. On the basis of several assumptions, such as time preferences and certain distributions, he characterizes the optimal mechanism as a sequence of prices increasing with each sale. This result somewhat rationalizes the exclusive restriction to dynamic pricing policies and impatient customers. But for a wide range of parametrizations, the benefit of dynamic pricing over a fixed posted price is minimal. Moreover, especially with low initial capacity, auctions are only close to optimal but may be preferable in practice due to their significantly greater robustness. The work of Yousefi et al. (2011) takes a similar direction. They show that in the presence of either uncertain supply or random demand, the seller's optimal allocation rule is a preannounced multistep priority scheme or markdown pricing schedule.

### 4.6 Uncertainty in terms of customers' individual valuation

Most authors assume that customers already know their individual valuation at the beginning of the selling horizon (as we did in Section 3.8, when discussing customers' knowledge about other customers' valuations). However, Swinney (2011) points out that this would be a gross simplification for some products like new and innovative items (like Nintendo's Wii or Apple's iPhone just after launch), media items (books, movies, etc.), or if the customers' requirements for the item are uncertain (the utility of snow skis for a potential weekend may depend on the weather). This list also includes tickets for sports and entertainment events whose value also depends on the weather but, moreover, on other factors that are difficult to predict like the quality of the event (Levin et al. 2010). Regarding the airline industry, business travellers may arrange their appointments during the selling horizon and therefore only know their travel needs quite late (see Koenigsberg et al. 2008 and Liu/Xiao 2008).
Regarding the mathematical models, the phenomenon outlined above is captured in various ways; they are justified by different potential applications and are also driven by mathematical tractability. For example, Prasad et al. (2011) and Yu et al. (2011) model customers who know their valuations after an advanced selling period. Swinney (2011) only considers customers who become aware of their valuation at a random point in time during the selling horizon. In his paper, this uncertainty is cited as the main reason why customers strategically delay their purchase. By initially offering the product at a lower introductory price, the seller should encourage customers to buy earlier; this allows him to better match supply and de-
mand using short-term replenishment (see Section 4.1). Koenigsberg et al. (2008) model business travellers who get to know their valuation just before departure. They find that for an intermediate capacity level, uncertainty about the arrival of the business segment will cause the firm to offer last-minute discounts to leisure travellers. The same applies when customers are uncertain whether the firm will offer last-minute discounts; then, in equilibrium, the firm will, with some probability, offer such discounts. Su (2009) discusses uncertainty in terms of customers' individual valuation as a possible cause of the behavioural phenomenon of buyer inertia. In a rather theoretical paper, Levin et al. (2010) consider customers who become aware of their valuation only upon consumption, that is, after the purchase, and prove the existence and uniqueness of equilibria as well as certain monotonicity properties. Levin et al. (2009) compare this to a random utility model, where a residual level of uncertainty in valuations corresponds to the random component in utility, and this uncertainty cannot be resolved by customers until the purchase is complete. Bhalla (2012) analyses a product whose value can either be high or low and is unknown to both the seller and the customers. Xu (2011) considers a unique setting where customers know their initial valuation, but continuously use the product after purchase and may suffer from a spontaneous drop in valuations at some random point in time. This is motivated by a product suddenly becoming outdated or out of fashion.

### 4.7 Demand for multiple product units

While most papers consider single-unit demand, an assumption that is appropriate for big ticket items such as cars or household appliances and consumer electronics, customers also request multiple units of certain types of products. One example given by Elmaghraby et al. (2008) is the B2B sector. They point out that both multi-unit demand and strategic behaviour are especially important when selling to business clients. An example from the B2C sector includes products with stable consumption patterns like household items and staple foods ( Su 2010b).

In this context, the question arises whether customers will spread their demand over several time periods or will always either request up to their maximum demand or nothing (all-ornothing). In a setting where business customers are present in the market right from the beginning and are interested in buying a certain number of units at a constant per-unit valuation for all units, Elmaghraby et al. (2008) show that it is always optimal to submit all-or-nothing bids. Su (2010b) focuses on consumer goods. If a customer buys more in a given period than he consumes, he can stock up for future consumption at an inventory holding cost. Moreover, customers incur a fixed cost for each purchase, for example for visiting the store. In his model, $\mathrm{Su}(2010 \mathrm{~b}$ ) shows that the seller will either set a constant fixed price or offer periodic price promotions. Such promotions are beneficial when frequent shoppers have a higher valuation than customers with a lower consumption rate.

### 4.8 Customers' and seller's risk preference

Another aspect that is taken into account by a couple of authors is customers' and seller's attitude towards risk. Especially when customers consider large purchases, that is, spend a significant fraction of their household budget on a new computer or major household appliance, they are typically risk-averse (see Liu/van Ryzin 2008). The same may apply to more individual or emotional goods like luxury fashion. The seller usually behaves risk-neutral because he is confronted with a large number of repeated decision problems. However, even if this argument may hold for a company as a whole, it does not necessarily apply to an individual decision maker. From a methodological point of view, risk-neutrality is justified by: (1) the ability to convert a stochastic revenue stream to the corresponding stream with expected values (via a costless insurance market) and (2) the existence of perfect capital markets to
convert a deterministic income stream to the most preferred one with the same present value. If any of these conditions are not met, risk-aversion should be taken into account.

Risk-averse customers are considered by Liu/van Ryzin (2008), Bansal/Maglaras (2009), Prasad et al. (2011), and Yousefi et al. (2011) in models with a risk-neutral seller. Regarding customers with identical risk preferences, Liu/van Ryzin (2008) show that it is not useful, on the basis of certain assumptions, to create an availability risk for risk-neutral customers. But at the same time, it may indeed be optimal to generate major shortages for highly risk-averse customers. Bansal/Maglaras (2009) extend the analysis of Liu and van Ryzin (2008). Besides some other modifications, they allow for customers to also differ in terms of their risk preferences. Several of the main insights still apply in this extended setting. When customers are risk-neutral, there is always an optimal policy with just two prices (see Section 3.5). When customers have a low risk-aversion, such a policy is near optimal. Prasad et al. (2011) analyse the impact of customers' risk preference on a retailer's advance selling strategy (see Section 4.3) and show analytically that the overall impact of a customer's risk averse preference is negative on the profit of a retailer who sells in advance. Thus, the retailer should sell in advance if the customers' expected valuation exceeds customers' expected surplus when not buying early at a certain threshold.
The recent papers by Wang/Hu (2011) and Yu et al. (2011) examine models with risk-neutral customers and a risk-averse seller. When demand follows uniform distribution and utility is a power function, Wang/Hu (2011) find that the optimal ordering quantity in their model is lower in the case of strategic customers than with myopic customers. Yu et al. (2011) incorporate a risk-averse seller in an extension to their model that focuses on the additional purchase option of advance selling (see Section 4.3). Specifically, the seller maximizes his expected profit subject to a so-called "risk-aversion constraint" ensuring that the realized profit is, almost surely, at least a given minimum value. They show under certain assumptions that advance selling can be more beneficial to a risk-averse seller than a risk-neutral one, since it can realize sales earlier and reduce the seller's exposure to demand risk.

### 4.9 Product quality choice

Generally, product quality may encompass everything that increases consumer utility, such as adding new features to a product, improving existing features, increasing the durability, or even performing marketing activities that increase customer awareness and the product's perceived value (see Kim/Swinney 2011). Unfortunately, a higher quality level often comes at the expense of increased manufacturing costs. The literature not only deals with the quality level of a single product but also analyses the effects of different quality levels for multiple products. For example, books are sold in several print formats such as hardcover, paperback, and large prints. Often, the more expensive hardcovers are introduced first and cheaper paperback versions are published later. Similarly, Apple offers its iPhone in multiple versions with varying levels of flash memory capacity. In these examples, offering the lower-end alternatives may cannibalize the higher-end product, but it helps to segment the market (see Parlatürk 2012).
Kim/Swinney (2011) consider the quality of their single product as a decision variable in their model. They find that with strategic customer behaviour, the firm should invest in a higher product quality, but with a smaller inventory than would be optimal with myopic customers. Prices may be higher or lower with strategic customers than with myopic customers. Interestingly, these results contrast with models that separate the product design decision (quality) from the operational decision (order quantity with demand uncertainty). In these models, strategic customer behaviour typically leads to lower quality and prices. The only paper that addresses two products with different quality levels is written by Parlatürk (2012). Although he
does not treat the quality levels as decision variables (because they are exogenously given to his model), Parlatürk (2012) gains some very interesting general insights: He shows that the loss due to strategic customer behaviour can be less with two product variants compared to the single-product variant, which hints that product variety can serve as a tool to deal with strategic customers. For this benefit, the additional product needs an inferior cost-to-quality ratio. Hence, a firm may find it attractive to sell a product variant that would be unprofitable otherwise. But product variety can also hurt profitability due to strategic customer behaviour in cases where customer impatience and firm costs are moderate.

## 5 Summary and Outlook

Over the past couple of years, there has been an increasing intensification of scientific activities regarding dynamic pricing with respect to strategic customer behaviour. Interestingly, the research builds on some quite early papers, such as the research carried out in the 1970s by Coase (1972) in the field of Economics, as well as the seminal paper by Besanko/Winston (1990). When compared to traditional dynamic pricing approaches assuming myopic customer behaviour, the various strategic models are generally more complex and the mathematical analysis is more demanding. Despite the considerable interest in the subject and the notable number of publications which exist today, no papers have been written to date which attempt to provide a systematic overview of the current status of research and position the existing publications within the wider context. This paper seeks to close this gap.
When examining the existing literature, the field initially appears to be fairly heterogeneous with a variety of different objects of study that are analysed using different model types. This is not least due to the fact that researchers from many scientific disciplines with different backgrounds, such as Operations Management, Marketing, and Economics, are dealing with dynamic pricing issues. Nevertheless, despite this heterogeneity, a more in-depth analysis shows that a number of specific criteria can be identified that in total can be used to formally separate one paper's object of study from another's. Based on these criteria, in this survey, we develop a comprehensive classification scheme for dynamic pricing approaches with strategic customers. Using this scheme, we then classify all existing papers systematically. This classification shows the main focus of the existing publications. At the same time, the variety of values for each of the criteria of the classification illustrates the breadth of the area of research. Furthermore, the paper systematises a number of relevant problem extensions that are mainly motivated from practical applications. Although each of them is only examined by a few authors, we consider them quite important in order to meet specific industries' requirements, as the generated insights are heavily dependant on these extensions. Such extensions are, for example, the consideration of advanced purchase options, like reservations, or riskaverse sellers and customers.

When analysing the literature as a whole, it turns out that, regardless of the breadth of existing research activities, there are certain relevant subjects that have as yet received little attention. As a result, the following focal points for future research have been identified:

- Given that there is no set of common generic basic models for dynamic pricing with strategic customers, current papers examine a variety of practical aspects, each with very specific models tailored especially to the various aspects of focus. This is not least due to the mathematically sophisticated depiction of strategic customer behaviour. However, this heterogeneity, not only regarding the problem settings under consideration but also the kind of used models or modelling techniques, makes it really difficult to immediately compare different authors' results. Thus, it is not surprising that authors themselves very seldom are able to compare their results with those obtained by other
authors. It would therefore be desirable to attain a set of predominant and widely accepted generic basic models integrating the key aspects of dynamic pricing with strategic customers to allow future work to directly build upon these models. From a modelling perspective in particular, this would make the field much more homogeneous.
- The development of such common generic basic models would also allow for the development of methodological papers, which concentrate purely on finding precise solution methods or more powerful heuristics. This would in turn help to make the resulting models more practically useful as is presently the case. In particular, although current publications in the field often derive interesting problem insights, most of them are based on stylized models, so that it is not obvious if and how they scale up to real-world problem sizes. Substantial progress has been made in this regard in recent years within the related area of revenue management. More precisely, there is a kind of basic revenue management problem setting and a small number of widely-accepted generic model types relying on both dynamic and linear programming, so that authors usually build their extensions and additional analysis on this common foundation. This has significantly simplified and driven the development of purely methodological papers, proposing several solution methods that allow application of the concepts in practice.
- Although most authors have direct practical applications in mind when incorporating extensions into their models and even refer to specific firms in the existing literature on dynamic pricing with strategic customers, there are only very few papers which combine an empirical with a price optimization section in their studies. Many papers are written by authors with an original background related to analytical modelling and mathematical optimization rather than to empirics and estimation. It would be quite promising to cover more research in this direction as this would further justify the practical relevance of the developed models and derived insights. Chan et al. (2009) also point out that there has not been much empirical application in the literature due to the complexity in computing and modelling of some of the equilibrium outcomes. However, growing interest in academic research is expected because of the recent development of computation and econometric techniques. Beyond that, more practical applications in different fields and sectors - as can be seen, for example, in the literature on dynamic structural models - would make the complex models of dynamic pricing with strategic customers more useful and workable for managers.
Despite the large number of relevant publications in recent years, the methodologically demanding field of dynamic pricing with strategic customers is still relatively new and continues to offer considerable opportunities for further research activities. Supported by its attractive positioning at the interface of different disciplines such as Operations Research, Marketing, and Economics, a variety of additional high-quality papers can be expected in the future.


## Comments

${ }^{1}$ Even as recently as 30 years ago there were only a few authors who dealt with the issue of intertemporal price discrimination, which is now regarded as the basis of dynamic pricing (e.g. Stokey 1979, Landsberger/Meilijson 1985, and Wilson 1988).
${ }^{2}$ See the following publications (sorted according to increasing losses of profit): Levin et al. (2009): $7 \%$, Zhang/Cooper (2008): $11 \%$, Mersereau/Zhang (2012): $11 \%$, Aviv/Pazgal (2008): 20\%, Levin et al. (2010): $20 \%$, Liu/van Ryzin (2008): $20 \%$, Nair (2007): $30 \%$, Besanko/Winston (1990): $50 \%$, and Cho et al. (2009): no details.
${ }^{3}$ Alongside the definition of the term "dynamic pricing" presented as such, there are also a variety of other definitions contained in publications with different objectives and different demands (e.g.

Gallego/van Ryzin 1994, p. 999, Kambil/Agrawal 2001, p. 16, Bitran/Caldentey 2003, p. 203, and Biller et al. 2005, p. 312). Dynamic pricing is also often understood to be a branch of revenue management. Reviews of the latter are provided, for example, by McGill/van Ryzin (1999), Tscheulin/Lindenmeier (2003), and Chiang et al. (2007). A comprehensive monograph is provided by Talluri/van Ryzin (2005).
${ }^{4}$ Note that some authors call the strategic customers we investigate "forward-looking consumers" (e.g. Aviv/Pazgal 2008, Swinney 2011, and Li/Zhang 2012). One of the first papers dealing with dynamic pricing with strategic customers by Besanko/Winston (1990), which is titled "Optimal price skimming by a monopolist facing rational consumers", does not use the term "strategic" at all.
${ }^{5}$ While literature on auctions in general is out of our scope, we consider papers including both auctions and dynamic pricing, either by applying both sales formats simultaneously (e.g. Gallien/Gupta 2007) or comparing them (e.g. Elmaghraby et al. 2009).
${ }^{6}$ See Chapter 7 (Su/Zhang 2009), Chapter 10 (Jerath et al. 2009), Chapter 12 (Aviv et al. 2009a), Chapter 13 (Aviv et al. 2009b), Chapter 14 (Cachon/Swinney 2009b), and Chapter 15 (Liu/van Ryzin 2009).
${ }^{7}$ In a Nash equilibrium, each player believes that the other players will choose the equilibrium strategies, and in equilibrium all players have an incentive to do so. More precisely, no market participant has an incentive to unilaterally deviate from its equilibrium decisions: Neither can the seller improve its profit nor can a consumer increase its individual consumer surplus. Each strategy of a risk neutral market participant is a best response to all other strategies in that equilibrium (e.g. Fudenberg/Tirole 1991 and Myerson 1997).
${ }^{8}$ Regarding this phenomenon, see also the seminal papers by Coase (1972) and Stokey (1981) as well as the follow up research from Bulow (1982), Conlisk et al. (1984), Gul et al. (1986), Ausubel/Deneckere (1989), Sobel (1991), DeGraba (1995), and from von der Fehr/Kühn (1995).
${ }^{9}$ This is also referred to as the "loss of early-adopter advantages" (e.g. Liu/Zhang 2012).
${ }^{10}$ Another option is used by Su (2007), who uses waiting costs instead of a discount factor to reflect the fact that customers have to forego a product at a certain time when purchasing later on. Gallego et al. (2008) directly attribute a lower valuation to customers in the second period.
${ }^{11}$ To the best of our knowledge, there is no research on problems where only the seller has a time preference.
${ }^{12}$ Note that in Table 3, we consider only information on exogenous aspects. This is because the players usually have so-called beliefs about endogenous values that they do not know, such as the seller's capacity choice (e.g. Liu/van Ryzin 2008) or future availability. The model is then developed describing the seller's and the customers' actions using these beliefs. Then, in a next step, the authors often focus on a rational expectations equilibrium (Muth 1961) that is dubbed a "self-fulfilling prophecy" by Yin et al. (2009) because the beliefs are rational in the sense that they are consistent with the actual outcome that materializes through the events that unfold based on having this particular belief. Moreover, in this equilibrium, no player has an incentive to unilaterally deviate from his equilibrium behaviour. Mak (2008) and Yin et al. (2009) do not follow this approach; we go into these papers when considering the relevant information aspect.
${ }^{13}$ Note that Allon/Bassamboo (2011) is not included in our classification, as it is based on exogenous prices. However, the authors describe endogenous pricing as a potential model extension.
${ }^{14}$ Exceptions are Gallego et al. (2008), Bansal/Maglaras (2009), and Borgs et al. (2011). In Borgs et al. (2011), customers need not care about this aspect because the seller announces prices upfront and has to choose prices such that the product does not sell out for any realization of demand. Similarly, the seller in Bansal/Maglaras (2009) communicates prices and availability rates upfront. In Gallego et al. (2008), customers directly base their considerations on a belief about the product's future availability which is consistent in equilibrium.
${ }^{15}$ See Section 4.6 for a discussion of some exceptions, that is, papers considering customers who do not know their individual valuations.
${ }^{16}$ In this context, the problem settings studied by Villas-Boas (2004) and Chen/Zhang (2009) are special. Here, the seller recognizes customers that already bought in a previous period and therefore can, under some conditions, infer their individual valuation.
${ }^{17}$ Analogously to market size, the seller's and customers' information on valuations is symmetric in most papers, with the exception of the studies mentioned in Endnote 14 where customers do not care about other customers' behaviour and therefore ignore their valuations.
${ }^{18}$ Note that there are very few exceptions, that is, some papers where the authors do not directly motivate their extension with specific practical evidence: for example, papers on risk preferences. However, one can easily think of real-world applications for which they are relevant.

## References

Allon G, Bassamboo A (2011) Buying from the babbling retailer? The impact of availability information on customer behavior. Manag Sci 57(4):713-726
Anderson CK, Wilson JG (2011) Name-your-own price auction mechanisms - Modeling and future implications. J Revenue Pricing Manag 10(1):32-39
Ausubel LM, Deneckere RJ (1989) Reputation in bargaining and durable goods monopoly. Econom 57(3):511-531
Aviv Y, Pazgal A (2008) Optimal pricing of seasonal products in the presence of forwardlooking consumers. Manuf Serv Oper Manag 10(3):339-359
Aviv Y, Vulcano G (2012) Dynamic list pricing. Forthcoming as Chapter 23 in: The Oxford Handbook of Pricing Management. Oxford University Press, Oxford
Aviv Y, Levin Y, Nediak M (2009a) Counteracting strategic consumer behavior in dynamic pricing systems. In: Netessine S, Tang CS (eds) Consumer-driven demand and operations management models. Springer, Dordrecht, pp 323-352
Aviv Y, Tang CS, Yin R (2009b) Mitigating the adverse impact of strategic waiting in dynamic pricing settings: A study of two sales machanisms. In: Netessine S, Tang CS (eds) Consumer-driven demand and operations management models. Springer, Dordrecht, p 353-370
Bansal M, Maglaras C (2009) Dynamic pricing when customers strategically time their purchase: Asymptotic optimality of a two-price policy. J Revenue Pricing Manag 8(1):4266
Bass FM (1969) A new product growth for model consumer durables. Manag Sci 15(5):215227
Besanko D, Winston WL (1990) Optimal price skimming by a monopolist facing rational consumers. Manag Sci 36(5):555-567
Bhalla M (2012) Dynamic pricing under social learning with strategic consumers. Working paper, Indian Institute of Management Bangalore
Bichler M, Kalagnanam JR, Katircioglu K, King AJ, Lawrence RD, Lee HS, Lin GY, Lu Y (2002) Applications of flexible pricing in business-to-business electronic commerce. IBM Syst J 41(2):287-302
Biller S, Chan LMA, Simchi-Levi D, Swann JL (2005) Dynamic pricing and the direct-tocustomer model in the automotive industry. Electron Commer Res 5(2):309-334
Bitran G, Caldentey R (2003) An overview of pricing models and revenue management. Manuf Serv Oper Manag 5(3):203-229
Biyalogorsky E (2009) Shaping consumer demand through the use of contingent pricing. In: Netessine S, Tang CS (eds) Consumer-driven demand and operations management models. Springer, Dordrecht, pp 423-434
Blattberg RC, Neslin SA (1990) Sales promotions - Concepts, methods, and strategies. Pren-tice-Hall, Englewood Cliffs, New Jersey

Borgs C, Candogan O, Chayes J, Lobel I, Nazerzadeh H (2011) Optimal multi-period pricing with service guarantees. Working paper, Microsoft Research, New England Lab, Cambridge
Bulow JI (1982) Durable-goods monopolists. J Polit Econ 90(21):314-323
Cachon GP, Swinney R (2009a) Purchasing, pricing, and quick response in the presence of strategic consumers. Manag Sci 55(3):497-511
Cachon GP, Swinney R (2009b) The impact of strategic consumer behavior on the value of operational flexibility. In: Netessine S, Tang CS (eds) Consumer-driven demand and operations management models. Springer, Dordrecht, pp 371-396
Cachon GP, Swinney R (2011) The value of fast fashion: Quick response, enhanced design, and strategic consumer behavior. Manag Sci 57(4):778-795
Chan LMA, Shen ZJM, Simchi-Levi D, Swann JL (2004) Coordination of pricing and inventory decisions: A survey and classification. In: Simchi-Levi D, Wu SD, Shen ZJM (eds) Handbook of quantitative supply chain analysis: Modeling in the e-business era. Kluwer, Boston, pp 335-392
Chan T, Kadiyali V, Xiao P (2009) Structural models of pricing. In: Rao VR (ed) Handbook of Pricing Research in Marketing. Edward Elgar Publishing, Northampton, pp 108-131
Chen Y, Zhang ZJ (2009) Dynamic targeted pricing with strategic consumers. Int J Ind Organ 27(1):43-50
Chevalier J, Goolsbee A (2009) Are durable goods consumers forward-looking? Evidence from college textbooks. Q J Econ 124(4):1853-1884
Chiang W-C, Chen JCH, Xu X (2007) An overview of research on revenue management: Current issues and future research. Int J Revenue Manag 1(1):97-128
Chintagunta P, Erdem T, Rossi PE, Wedel M (2006) Structural modeling in marketing: Review and assessment. Mark Sci 25(6):604-616
Cho M, Fan M, Zhou J-P (2009) Strategic consumer response to dynamic pricing of perishable products. In: Netessine S, Tang CS (eds) Consumer-driven demand and operations management models. Springer, Dordrecht, pp 435-458
Coase RH (1972) Durability and monopoly. J Law Econ 15(1):143-149
Conlisk J, Gerstner E, Sobel J (1984) Cyclic pricing by a durable goods monopolist. Q J Econ 99(3):489-505
Dasu S, Tong C (2010) Dynamic pricing when consumers are strategic: Analysis of a posted pricing scheme. Eur J Oper Res 204(3):662-671
DeGraba P (1995) Buying frenzies and seller-induced excess demand. Rand J Econ 26(2):331-342
Diller H (2008) Preispolitik. 4th edn, Kohlhammer, Stuttgart
Elmaghraby W, Keskinocak P (2003) Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. Manag Sci 49(10):1287-1309
Elmaghraby W, Gülcü A, Keskinocak P (2008) Designing optimal preannounced markdowns in the presence of rational customers with multiunit demand. Manuf Serv Oper Manag 10(1):126-148
Elmaghraby W, Lippman SA, Tang CS, Yin R (2009) Will more purchasing options benefit customers? Prod Oper Manag 18(4):381-401
Erdem T, Imai S, Keane MP (2003) Brand and quantity choice dynamics under price uncertainty. Quant Mark and Econ 1(1):5-64
Erdem T, Keane MP, Öncü TS, Strebel J (2005) Learning about computers: An analysis of information search and technology choice. Quant Mark Econ 3(3):207-247
Etzioni O, Knoblock CA, Tuchinda R, Yates A (2003) To buy or not to buy: Mining airfare data to minimize ticket purchase price. Proceedings of the 9th ACM SIGKDD Interna-
tional Conference on Knowledge Discovery and Data Mining, ACM Press, New York, pp 119-128
Fibich G, Gavious A, Lowengart O (2003) Explicit solutions of optimization models and differential games with nonsmooth (asymmetric) reference-price effects. Oper Res 51(5):721-734
Filene's Basement (2012) The birth of a legend - our story. http://www.filenesbasement.com/our_story.php. Accessed 23 Aug 2012
Fudenberg D, Tirole J (1991) Game theory. MIT Press, Cambridge
Gallego G, van Ryzin G (1994) Optimal dynamic pricing of inventories with stochastic demand over finite horizons. Manag Sci 40(8):999-1020
Gallego G, Phillips R, Sahin O (2008) Strategic management of distressed inventory. Prod Oper Manag 17(4):402-415
Gallien J (2006) Dynamic mechanism design for online commerce. Oper Res 54(2):291-310
Gallien J, Gupta S (2007) Temporary and permanent buyout prices in online auctions. Manag Sci 53(5):814-833
Gönsch J, Klein R, Steinhardt C (2009) Dynamic Pricing - State-of-the-Art. Z Betriebswirtschaft 79(Special Issue 3):1-40
Gönsch J, Steinhardt C (2012) Using dynamic programming decomposition for revenue management with opaque products. Working paper, University of Augsburg
Greenleaf EA (1995) The impact of reference price effects on the profitability of price promotions. Mark Sci 14(1):82-104
Gul F, Sonnenschein H, Wilson R (1986) Foundations of dynamic monopoly and the Coase conjecture. J Econ Theory 39(1):155-190
Helson H (1964) Adaptation-level theory: An experimental and systematic approach to behavior. Harper \& Row, New York
Hinz O, Hann I-H, Spann M (2011) Price discrimination in e-commerce? An examination of dynamic pricing in name-your-own-price markets. MIS Quarterly 35(1):81-98
Ho TH, Su X (2009) Strategic pricing response and optimization in operations management. In: Rao VR (ed) Handbook of Pricing Research in Marketing. Edward Elgar Publishing, Northampton, pp 557-580
Huang T, van Mieghem JA (2011) Are strategic customers willing to visit a website when their clickstreams are tracked? Working paper, Kellogg School of Management, Northwestern University, Evanston
Jerath K, Netessine S, Veeraraghavan SK (2009) Selling to strategic customers: Opaque selling strategies. In: Netessine S, Tang CS (eds) Consumer-driven demand and operations management models. Springer, Dordrecht, pp 253-300
Jerath K, Netessine S, Veeraraghavan SK (2010) Revenue management with strategic customers: Last-minute selling and opaque selling. Manag Sci 56(3):430-448
Kahneman D, Tversky A (1979) Prospect theory: An analysis of decision under risk. Econom 47(2):263-292
Kambil A, Agrawal V (2001) The new realities of dynamic pricing. Accenture Outlook J 2001(2):15-21
Kim S-H, Swinney R (2011) Product quality choice and inventory risk with strategic consumers. Working paper, Yale School of Management, Yale University, New Haven
Klein R, Steinhardt C (2008) Revenue Management - Grundlagen und mathematische Methoden. Springer, Berlin Heidelberg
Koenigsberg O, Muller E, Vicassim NJ (2008) EasyJet pricing strategy: Should low-fare airlines offer last-minute deals? Quant Mark Econ 6(3):279-297
Kopalle PK, Rao AG, Assunção JL (1996) Asymmetric reference price effects and dynamic pricing policies. Mark Sci 15(1):60-85

Lai G, Debo LG, Sycara K (2010) Buy now and match later: Impact of posterior price matching on profit with strategic consumers. Manuf Serv Oper Manag 12(1):33-55
Landsberger M, Meilijson I (1985) Intertemporal price discrimination and sales strategy under incomplete information. Rand J Econ 16(3):424-430
Levin Y, McGill J, Nediak M (2009) Dynamic pricing in the presence of strategic consumers and oligopolistic competition. Manag Sci 55(1):32-46
Levin Y, McGill J, Nediak M (2010) Optimal dynamic pricing of perishable items by a monopolist facing strategic consumers. Prod Oper Manag 19(1):40-60
Li C, Zhang F (2012) Advance demand information, price discrimination, and pre-order strategies. Working paper, School of Business, University of Connecticut, Storrs
Li J, Granados N, Netessine S (2011) Are consumers strategic? Structural estimation from the air-travel industry. Working paper, Wharton School, University of Pennsylvania, Philadelphia
Liu Q, van Ryzin G (2008) Strategic capacity rationing to induce early purchases. Manag Sci 54(6):1115-1131
Liu Q, van Ryzin G (2009) Capacity rationing with strategic customers. In: Netessine S, Tang CS (eds) Consumer-driven demand and operations management models. Springer, Dordrecht, pp 253-300
Liu Q, Xiao W (2008) Selling to heterogeneous customers with uncertain valuations under returns policies. Working paper, Hong Kong University of Science and Technology
Liu Q, Zhang D (2012) Dynamic pricing competition with strategic customers under vertical product differentiation. Working paper, Hong Kong University of Science and Technology
Mak V (2008) Disclosing inventory to mitigate the impact of strategic waiting on profits. Working paper, Hong Kong University of Science and Technology
Mak V, Rapoport A, Gisches EJ (2012) Competitive dynamic pricing with alternating offers: Theory and experiment. Games Econ Behav 75(1):250-264
Mantin B, Granot D (2010) Dynamic pricing in the presence of store-switching consumers. Working Paper, Sauder School of Business, University of British Columbia, Vancouver
McGill JI, van Ryzin GJ (1999) Revenue management: Research overview and prospects. Transp Sci 33(2):233-256
Melnikov O (2000) Demand for differentiated durable products: The case of the U.S. computer printer market. Working paper, Yale University, New Haven
Mersereau AJ, Zhang D (2012) Markdown pricing with unknown fraction of strategic customers. Manuf Serv Oper Manag 14(3):355-370
Mulpuru S, Hult P (2008) The future of online secondary ticketing. Forrester Research, Cambridge
Muth JF (1961) Rational expectations and the theory of price movements. Econom 29(3):315-335
Myerson RB (1997) Game theory: Analysis of conflict. Havard University Press, Cambridge
Nair H (2007) Intertemporal price discrimination with forward-looking consumers: Application to the US market for console video-games. Quant Mark Econ 5(3):239-292
Netessine S, Tang CS (2009) Consumer-driven demand and operations management models. Springer, Dordrecht
Osadchiy N, Vulcano G (2010) Selling with binding reservations in the presence of strategic consumers. Manag Sci 56(12):2173-2190
Parlaktürk AK (2012) The value of product variety When selling to strategic consumers. Manuf Serv Oper Manag 14(3):371-385
Pashigian BP, Bowen B (1991) Why are products sold on sale?: Explanations of pricing regularities. Q J Econ 106(4):1015-1038
Phillips RL (2005) Pricing and revenue optimization. Stanford University Press, Stanford

Popescu I, Wu Y (2007) Dynamic pricing strategies with reference effects. Oper Res 55(3):413-429
Prasad A, Stecke KE, Zhao X (2011) Advance selling by a newsvendor retailer. Prod Oper Manag 20(1):129-142
Roselius T (1971) Consumer rankings of risk reduction methods. J Mark 35(1):56-61
Shen ZJM, Su X (2007) Customer behavior modeling in revenue management and auctions: A review and new research opportunities. Prod Oper Manag 16(6):713-728
Shoemaker RW, Shoaf FR (1975) Behavioral changes in the trial of new products. J Consum Res 2(2):104-109
Simon H, Faßnacht M (2009) Preismanagement: Strategie - Analyse - Entscheidung - Umsetzung. 3rd edn, Gabler, Wiesbaden
Sobel J (1991) Durable goods monopoly with entry of new consumers. Econom 59(5):14551485
Song I, Chintagunta PK (2003) A micromodel of new product adoption with heterogeneous and forward-looking consumers: Application to the digital camera category. Quant Mark Econ 1(4):371-407
Stokey NL (1979) Intertemporal price discrimination. Q J Econ 93(3):355-371
Stokey NL (1981) Rational expectations and durable goods pricing. Bell J Econ 12(1):112128
Spann M, Skiera B, Schäfers B (2004) Measuring individual frictional costs and willingness-to-pay via name-your-own-price mechanisms. J Interact Mark 18(4):22-36
Su X (2007) Intertemporal pricing with strategic customer behavior. Manag Sci 53(5):726741
Su X (2009) A model of consumer inertia with applications to dynamic pricing. Prod Oper Manag 18(4):365-380
Su X (2010a) Optimal pricing with speculators and strategic consumers. Manag Sci 56(1):2540
Su X (2010b) Intertemporal pricing and consumer stockpiling. Oper Res 58(4):1133-1147
Su X, Zhang F (2008) Strategic customer behavior, commitment, and supply chain performance. Manag Sci 54(10):1759-1773
Su X, Zhang F (2009) Strategic customer behavior and the benefit of decentralization. In: Netessine S, Tang CS (eds) Consumer-driven demand and operations management models. Springer, Dordrecht, pp 177-204
Sun B (2005) Promotion effect on endogenous consumption. Mark Sci 24(3):430-443
Sun B, Neslin SA, Srinivasan K (2003) Measuring the impact of promotions on brand switching when consumers are forward looking. J Mark Res 40(4):389-405
Swinney R (2011) Selling to strategic consumers when product value is uncertain: The value of matching supply and demand. Manag Sci 57(10):1737-1751
Talluri KT, van Ryzin GJ (2005) The theory and practice of revenue management. Springer, New York
Tang CS (2010) A review of marketing - operations interface models: From co-existence to coordination and collaboration. Int J Prod Econ 125(1):22-40
Tereyağoğlu N, Veeraraghavan S (2012) Selling to conspicuous consumers: Pricing, production, and sourcing decisions. Forthcoming: Manag Sci, doi: $10.1287 / \mathrm{mnsc} .1120 .1545$
Tscheulin DK, Lindenmeier J (2003) Yield-Management - Ein State-of-the-Art. Z Betriebswirtschaft 73(6):629-662
Villas-Boas JM (2004) Price cycles in markets with customer recognition. Rand J Econ 35(3):486-501
von der Fehr N-HM, Kühn K-U (1995) Coase versus Pacman - Who eats whom in the dura-ble-goods monopoly. J Polit Econ 103(4):785-812

Wang T, Hu Q (2011) Risk-averse newsvendor model with strategic consumers. Paper presented at the 23rd Chinese Control and Decision Conference, Mianyang
Wilson CA (1988) On the optimal pricing policy of a monopolist. J Polit Econ 96(1):164-176 Xu FZ (2011) Optimal best-price policy. Int J Ind Organ 29(5):628-643
Yin R, Aviv Y, Pazgal A, Tang CS (2009) Optimal markdown pricing implications of inventory display formats in the presence of strategic customers. Manag Sci 55(8):1391-1408
Yousefi SR, Rui H, Whinston A (2011) Optimal markdown and priority pricing with uncertainty: retailers burn commodities. Working paper, The University of Texas at Austin
Yu M, Kapuscinski R, Ahn H-S (2011) Advance selling with interdependent customer valuations. Working paper, Hong Kong University of Science and Technology
Zhang D, Cooper WL (2008) Managing clearance sales in the presence of strategic customers. Prod Oper Manag 17(4):416-431
Zhou C, Wu Y (2011) Study on revenue management considering strategic customer behavior. J Serv Sci Manag 4(4):507-512

