Revenue management with flexible products: The value of flexibility and its incorporation into DLP-based approaches

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Abstract

A major benefit of flexible products is that they allow for supply-side substitution even after they have been sold. This helps improve capacity utilization and increase the overall revenue in a stochastic environment. As several authors have shown, flexible products can be incorporated into the well-known deterministic linear program (DLP) of revenue management's capacity control. In this paper, we show that flexible products have an additional "value of flexibility" due to their supply-side substitution possibilities, which can be captured monetarily. However, the DLP-based approaches proposed so far fail to capture this value and, thus, steadily undervalue flexible products, resulting in lower overall revenues. To take the full potential of flexible products into account, we propose a new approach that systematically increases the revenues of flexible products when solving the DLP and performing capacity control. A mathematical function of variables available during the booking horizon represents this artificial markup and adapts dynamically to the current situation. We determine the function's parameters using a standard simulation-based optimization method. Numerical experiments show that the benefits of the new approach are biggest when low value demand arrives early. Revenues are improved by up to 5% in many settings.

Keywords: Revenue Management, Flexible Products, DLP-based Approaches

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1 Introduction

In this paper, we reconsider the well-known revenue management problem of optimal capacity control with flexible products. Flexible products allow the provider to decide on the utilized resources sometime after selling the product. Gallego and Phillips (2004) were the first to introduce the problem to the academic literature. They were motivated by the popularity of these supply-side substitution opportunities and the high practical relevance resulting from them, for example, in the context of airlines, hotels, and cruise lines. Moreover, Gallego et al. (2004) incorporated flexible products into the common dynamic programming approach for network revenue management. Similar to the standard setting, the dynamic program is computationally intractable even for modest problem sizes due to the multidimensional state space that must be considered. Researchers have therefore investigated different types of approximations in a considerable number of follow-up papers. The most prominent approximations employ a deterministic linear program (DLP), which is quite common in practical applications. It is obtained by simply replacing any uncertainty in the dynamic program with expected values.

As we demonstrate in this paper, even though many researchers have followed DLP-based approaches, the straightforward extension of the DLP does not take the full potential of flexible products into consideration and has additional drawbacks compared to its application in the standard setting. More precisely, our contributions are as follows: First, given the dynamic program of Gallego et al. (2004), we analytically isolate the additional monetary “value of flexibility” that comes along with acceptance of flexible requests. Second, we show that none of the DLP-based approaches proposed so far considers this value. Therefore, flexible products’ benefits are systematically underestimated and the resulting control mechanisms are too restrictive regarding the acceptance of flexible requests. Third, we propose a new and straightforward DLP-based approach that avoids the strict preference of regular products and, by using simulation-based optimization, better incorporates the benefits of flexible products. An extensive numerical study demonstrates the applicability of this approach and shows that, in most settings, it significantly outperforms existing approaches in terms of the overall achievable revenue.

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The remainder of this paper is structured as follows: In Section 2, we accurately restate the revenue management problem of optimal capacity control with flexible products, including the assumptions made. Furthermore, we carve out the relevance of the problem by providing examples from various industries and review the relevant scientific literature. In Section 3, we summarize the standard models for network revenue management with flexible products. Based on this, we begin Section 4 with the analytical derivation and investigation of the value of flexibility in the dynamic program. We then turn to the DLP-based approximations and show why they have additional shortcomings with regard to flexible products. In Section 5, we present our new, improved approach and investigate its performance computationally in Section 6. In Section 7, we discuss the potential limitations of the chosen approach, as well as the assumptions, and conclude the paper.

2 Problem statement, practical relevance, and related literature

In this section, we provide an overview of revenue management with flexible products. We first state the problem of capacity control with flexible products in detail. Using various examples from different industries, we then show the problem's relevance in practice. Finally, we extensively review the relevant scientific literature.

2.1 Problem statement

As a result of price discrimination, a firm offers differently priced products that are provided using a number of shared resources with a fixed capacity. Customers arrive successively and stochastically throughout a fixed booking horizon with each customer requesting one unit. The requested product is independent of the available products and of other customers (the well-known independent demand assumption). Service provision occurs at the end of the booking horizon. Any capacity remaining at the end of the booking horizon is worthless and overbooking of the given resources' capacity is not allowed. Besides regular products, each of which requires one unit of capacity from one or more affected resources, there are also flexible products, whose final resources the firm can decide at the end of the booking horizon – just before service provision. More precisely, the firm selects an execution mode from a pre-specified set of modes, in which each execution mode – like regular products – is associated with the consumption of one or more resources. From a customer's perspective, flexible products are usually inferior due to their inherent uncertainty (see, e.g., Gallego and Phillips 2004). Accordingly, it is assumed that a flexible product is cheaper than a comparable regular product. Revenue management's capacity control continuously addresses the following decision problem throughout the booking horizon: Should the firm, upon a customer's arrival, accept his product request and collect the associated revenue, or should it reject this request and retain its capacity to accept a higher value request that might arrive in the future? An appropriate demand forecast is available, but forecast errors can bias it.

2.2 Flexible products in practice

The problem of revenue management with flexible products is highly relevant in practice. Many capacity providers operate in competitive markets and rely heavily on revenue management techniques to stay profitable. To further boost revenues, they increasingly turn to flexible products, as the following examples show.

Tour operators rank among the most prominent providers of flexible products. Most of them offer a travel roulette. This product hides the exact hotel during the booking process, but customers are aware of the category and area. They are often only informed about their hotel on their arrival at the destination airport. For example, TUI offers RIU package holidays (www.riu.com) that only allow the customer to choose the hotel category and the region. Neckermann (www.neckermann-reisen.de) has an almost identical product.

Cruise lines use flexible products for price discrimination. As passengers spend a lot of time together onboard and are very likely to talk about the prices they paid, different prices for basically the same product would be perceived as unfair. However, if faced with a flexible product, customers are more likely to acknowledge the discount as justified due to the uncertainty involved, which many customers simply find unacceptable. For example, AIDA Cruises (www.aida.com) offers the product "JUST AIDA," which comprises a flexible travel time within a specified time window, or one of several pre-specified ships with different routes, for example, the eastern and western Mediterranean. These details are communicated to the passenger two weeks before departure.

Cargo customers are usually not interested in their freight's specific route. Thus, the transporting company has a certain degree of freedom regarding when and how to assign requests to specific flights. For example, Lufthansa Cargo (www.lufthansa-cargo.com) offers the product "td.Pro," that allows a customer to only specify the time frame for guaranteed pickup.

Software providers that support companies selling flexible products and services have recently come on the market. An example is SigmaZen (www.sigmazen.de), which offers I-DEAL. This software is compatible with established computer reservation systems such as Amadeus or Navitaire. I-DEAL is based on the idea that a company provides a discount for a customer-specific degree of flexibility. The more flexible a customer is, the higher the discount.

2.3 Literature review

In the following, we review the academic literature on revenue management's capacity control and on flexible products. We provide a brief but broad overview and delve deeper where the research is directly related to our work.

Since academic research on revenue management started some 30 years ago, a considerable amount of work has been published on revenue management models that allow for the automation of capacity control. Overviews can be found in the textbooks by Talluri and van Ryzin (2004a) as well as Phillips (2005). Modeling the provider's decision problem of optimal capacity control as a stochastic dynamic program (DP) has become widely accepted as the de facto standard formulation. Furthermore, various properties of optimal control policies have been derived in respect of the special case of only one resource (see, e.g., Lee and Hersh 1993 and Subramanian et al. 1999). However, as soon as multiple resources are considered, it is well known that only the most basic properties still hold. Moreover, the dynamic program for such a resource network is computationally only tractable in very small instances. Thus, heuristic approaches have been developed that approximate the DP. The simplest and most popular approximation is the DLP (see, e.g., Talluri and van Ryzin 1998 for the standard formulation), which is justified by its solution converging to that of the DP. In practice, the DLP is regularly recalculated, using updated demand and capacity data throughout the booking horizon (see, e.g., Talluri and van Ryzin 2004a, Chapter 3.3.1). Besides the DLP, various other approximations have been developed (see, e.g., Adelman 2007 for an approximate dynamic programming approach and Hung and Chen 2013 for a scenario tree approach).

Deriving bid prices in order to decide on the acceptance of requests is the most popular way to operationalize approximations; this paper thus also focuses on this approach. These bid prices represent a threshold price of one unit of capacity of each resource, which reflects the opportunity cost of selling this unit. Accordingly, a product request is accepted if the revenue exceeds the sum of the bid prices of the resources that that product uses. The standard bid price control was initially proposed by Smith and Penn (1988) and Simpson (1989). Talluri and van Ryzin (1998, 1999) subsequently examined it in detail. Another – but today less popular – way of operationalizing some of the approximations are booking limits that explicitly specify the maximum amount of each product to be sold. These limits are regularly updated throughout the booking horizon (see, e.g., Bertsimas and de Boer 2005 and Haerian et al. 2006 for corresponding algorithms). Booking limits are usually nested, meaning that higher-class products may access the capacity of lower-class products. However, nesting with regard to multiple resources is often too complicated (see, e.g., Smith et al. 1992 for the well-known virtual nesting approach).

Over the last couple of years, two important trends in revenue management can be identified. First, settings with arbitrary customer behavior that do not assume independent demand are considered (see, e.g., Talluri and van Ryzin 2004b and Liu and van Ryzin 2008). Second, and more important to us, the innovative product concepts flexible products, upgrades, and opaque products have been integrated into the classical network revenue management setting to allow for some supply-side substitution. Gallego and Phillips (2004) introduced flexible products in a simplified setting with two regular products that correspond to two resources. Gallego et al. (2004) extended the problem to an arbitrary number of products and resources. Their DP formulation became standard for revenue management with flexible products and is in line with the problem setting considered in this paper (see Sections 2.1 and 3.1). In a previous work (Petrick et al. 2010), we consider a corresponding DLP approximation and propose practical capacity control techniques in order to exploit the firm's flexibility over time. In a subsequent study (Petrick et al. 2012), we show how flexible products can help mitigate the demand uncertainty associated with inaccurate demand forecasts. The general concept of modeling supply-side flexibility and exploiting it over time is also applicable in several other fields. Examples include the air cargo industry (see, e.g., Bartodziej and Derigs 2004, as well as Bartodziej et al. 2007), the broadcasting industry (see, e.g., Müller-Bungart 2007, as well as Kimms and Müller-Bungart 2007), and make-to-order manufacturing environments (see, e.g., Spengler et al. 2007). Moreover, upgradeable products can be seen as a special case of flexible products, as their multiple, but hierarchically ordered, execution modes also provide flexibility. Gallego and Stefanescu (2009), Steinhardt and Gönsch (2012), as well as Gönsch et al. (2013) have recently researched appropriate capacity control approaches.

Moreover, opaque (or probabilistic) products for which the firm decides the execution mode immediately after sale are also closely related. Recently, this type of product has been intensively researched from an economics and marketing perspective. Mang et al. (2012) empirically show that a lower price, a higher self-selected level of flexibility, and a higher search intensity increase the probability of purchase. Fay and Xie (2010) analyze how selling these products improves profits by inducing additional low value demand and avoiding excess cannibalization, especially when compared to advance selling. Post (2010) presents a pricing heuristic for opaque products, while Post and Spann (2012) illustrate their successful implementation of such products at Lufthansa's subsidiary Germanwings. Again, from a revenue management perspective, Chen et al. (2010), as well as Gönsch and Steinhardt (2013) formulate corresponding capacity control approaches. If – unlike the aforementioned product types – supply-side substitution occurs without the customer's consent, a monetary compensation may be necessary. For example, Ge et al. (2010), as well as Huang et al. (2013) research transferring passengers to parallel flights to mitigate overbooking.

From a methodological point of view, the application of simulation-based optimization to capacity control is also related to our work. Klein (2007) first used it in the context of bid prices for the traditional revenue management problem, followed by Meissner and Strauss (2012), who used it in a setting with customer choice, and Volling et al. (2012) who employed it for make-to-order revenue management. Graf and Kimms (2011, 2013) propose simulation-based approaches for capacity control in airline alliances.

3 Basic model formulations

In this section, we briefly summarize the standard models for network revenue management with flexible products from the literature. First, we introduce the relevant notation. We then restate the standard DP for revenue management with flexible products and specify the optimal control policy. Finally, we state the corresponding DLP approximation.

3.1 Notation

Formalizing the problem statement from Section 2.1, we assume that a firm offers several regular products $\mathcal{I} = \{1,\ldots,n^{reg}\}$ based on a set of resources $\mathcal{H} = \{1,\ldots,m\}$, where r_i^{reg} denotes the revenue of a product $i \in \mathcal{I}$. The vector $\boldsymbol{a_i} = (a_{1i},\ldots,a_{mi})$ denotes a product's capacity consumption, with $a_{hi} = 1$ if product i requires resource h and $a_{hi} = 0$ otherwise. In addition, the firm offers the flexible products $\mathcal{J} = \{1,\ldots,n^{flex}\}$. By selling a flexible product $j \in \mathcal{J}$ with revenue r_j^{flex}, the firm guarantees that the customer will be assigned to one of the execution modes $\mathcal{M}_j \subseteq \mathcal{I}$, which – in terms of resource consumption – we assume to be a subset of the existing regular products without loss of generality. The resources' (remaining) capacity is denoted by the vector $\boldsymbol{c} = (c_1,\ldots,c_m)$. Whereas the sale of a regular product immediately reduces capacity of one or more resources, the sale of a flexible product does not because its execution mode is only decided at the end of the selling horizon. Nonetheless, the firm has to ensure that the remaining capacity is sufficient to satisfy the accepted requests. Thus, for each flexible product j, the number of already accepted requests (commitments) is summed by the parameter y_j^a. The vector of the commitments is denoted by $\boldsymbol{y^a} = \left(y_1^a, \ldots, y_{n^{flex}}^a\right)$ and $\boldsymbol{e_j}$ refers to the j-th standard basis vector in $\mathbb{R}^{n^{flex}}$.

The booking horizon can be sufficiently discretized into T micro periods. The periods are numbered forward in time, and in each period t there is, at most, one customer arrival (see, e.g., Lee and Hersh 1993). The probabilities of an arrival in period t are denoted by $\lambda_i^{reg}(t)$ and $\lambda_j^{flex}(t)$. Consequently, the probability of there being no incoming request is $\lambda_0(t) = 1 - \sum_{i \in \mathcal{I}} \lambda_i^{reg}(t) - \sum_{j \in \mathcal{J}} \lambda_j^{flex}(t)$. Table 1 summarizes the notation used throughout this section. Note that when presenting mathematical formulations in the following two subsections, we introduce additional notation that is specific to these formulations, but is already included in the table.

$h \in \mathcal{H} = \{1, \ldots, m\}$: resources	$\lambda_j^{flex}(t)$: arrival probability of product j in period t
c_h : capacity of resource h	$\lambda_0(t)$: probability of no arrival in period t
$\boldsymbol{c} = (c_1, \ldots, c_m)$: vector of capacity	$V_t\left(\boldsymbol{c}, \boldsymbol{y^a}\right)$: optimal future expected revenue in state $\left(\boldsymbol{c}, \boldsymbol{y^a}\right)$ and period t
$i \in \mathcal{I} = \left\{1, \ldots, n^{reg}\right\}$: regular products	$\mathcal{Z}$: set of feasible states
r_i^{reg} : revenue of product i	$\Delta_i^{reg} V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y^a}\right)$: opportunity cost of acceptance of regular product i in state $\left(\boldsymbol{c}, \boldsymbol{y^a}\right)$ and period t
a_{hi} : capacity consumption of product i on resource h	$\Delta_j^{flex} V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y^a}\right)$: opportunity cost of acceptance of flexible product j in state $\left(\boldsymbol{c}, \boldsymbol{y^a}\right)$ and period t
$\boldsymbol{a_i} = (a_{1i}, \ldots, a_{mi})$: vector of capacity consumption of product i	D_{it}^{reg} : expected demand-to-come of product i
$j \in \mathcal{J} = \left\{1, \ldots, n^{flex}\right\}$: flexible products	D_{jt}^{flex} : expected demand-to-come of product j
r_j^{flex} : revenue of product j	x_i : future capacity allocation to product i
$\mathcal{M}_j$: execution modes of product j	y_{ji} : future capacity allocation to product j in execution mode i
y_j^a : number of commitments for product j	y_{ji}^a : temporary capacity allocation to product j in execution mode i for accepted requests
$\boldsymbol{y^a} = \left(y_1^a, \ldots, y_{n^{flex}}^a\right)$: vector of commitments	V^{DLP} : objective function of the DLP
$\boldsymbol{e_j}$: j-th standard basis vector in $\mathbb{R}^{n^{flex}}$	
$t \in \{1, \ldots, T\}$: micro periods	
$\lambda_i^{reg}(t)$: arrival probability of product i in period t	

Table 1: Notation introduced in Section 3

3.2 Dynamic programming formulation and optimal policy

The firm's decision problem of optimal capacity control is formulated exactly by the stochastic dynamic program (DP) given by the following Bellman equation (see Gallego et al. 2004):

$$V_t\left(\boldsymbol{c}, \boldsymbol{y^a}\right) = \sum_{i \in \mathcal{I}} \lambda_i^{reg}(t) \cdot \max\left\{r_i^{reg} + V_{t+1}\left(\boldsymbol{c} - \boldsymbol{a_i}, \boldsymbol{y^a}\right), V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y^a}\right)\right\}$$

$$+\sum_{j\in\mathcal{J}}\lambda_j^{flex}(t)\cdot\max\left\{r_j^{flex}+V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}+\boldsymbol{e_j}\right),V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right)\right\}$$
$$+\lambda_0(t)\cdot V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right) \qquad (1)$$

where $V_t\left(\boldsymbol{c},\boldsymbol{y^a}\right)$ denotes the optimal expected revenue-to-go for $t=1,\ldots,T$ and $\left(\boldsymbol{c},\boldsymbol{y^a}\right)\in\mathcal{Z}$. The set $\mathcal{Z}$ describes the feasible states; that is, all states where the remaining capacity is non-negative and sufficient to satisfy all accepted flexible requests (see Gallego et al. 2004 for a formal definition). The boundary conditions are $V_t\left(\boldsymbol{c},\boldsymbol{y^a}\right)=-\infty$ if $\left(\boldsymbol{c},\boldsymbol{y^a}\right)\notin\mathcal{Z}$ and $V_T\left(\boldsymbol{c},\boldsymbol{y^a}\right)=0$ if $\left(\boldsymbol{c},\boldsymbol{y^a}\right)\in\mathcal{Z}$.

In an optimal policy, customer requests for products i and j are accepted if and only if the revenue associated with the request is not less than its opportunity cost $\Delta_i^{reg}V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right)$ and $\Delta_j^{flex}V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right)$, respectively. More formally, regular products i are offered for sale if and only if

$$r_i^{reg}\geq\Delta_i^{reg}V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right) \qquad (2)$$

with opportunity cost defined as

$$\Delta_i^{reg}V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right):=V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right)-V_{t+1}\left(\boldsymbol{c}-\boldsymbol{a_i},\boldsymbol{y^a}\right). \qquad (3)$$

Flexible products j are offered if and only if

$$r_j^{flex}\geq\Delta_j^{flex}V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right) \qquad (4)$$

with opportunity cost

$$\Delta_j^{flex}V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right):=V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}\right)-V_{t+1}\left(\boldsymbol{c},\boldsymbol{y^a}+\boldsymbol{e_j}\right). \qquad (5)$$

3.3 DLP approximation

Let D_{it}^{reg} and D_{jt}^{flex} denote the expected demand-to-come of products i and j, respectively, aggregated from the current point in time t to the end of the booking horizon. Furthermore, decision variables are introduced that reflect the units of remaining capacity reserved for the different products. Specifically, the decision variables x_i denote the capacity allocation to the regular products i with respect to future requests. Regarding flexible products, the capacity allocations y_{ji} reflect the capacity reserved for future requests for product j in execution mode $i\in\mathcal{M}_j$. In addition, y_{ji}^a represents a temporary capacity allocation for requests that have already been accepted and are thus contained in y_j^a. Now, the resulting optimization model (DLP) is given as follows (see Petrick et al. 2012):

$$\text{Maximize } V^{DLP} = \max \sum_{i \in \mathcal{I}} r_i^{reg} \cdot x_i + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{M}_j} r_j^{flex} \cdot y_{ji} \tag{6}$$

subject to

$$\sum_{i \in \mathcal{M}_j} y_{ji}^a = y_j^a \qquad \text{for all } j \in \mathcal{J} \tag{7}$$

$$\sum_{i \in \mathcal{I}} a_{hi} \cdot x_i + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{M}_j} a_{hi} \cdot \left(y_{ji}^a + y_{ji} \right) \le c_h \qquad \text{for all } h \in \mathcal{H} \tag{8}$$

$$x_i \le D_{it}^{reg} \qquad \text{for all } i \in \mathcal{I} \tag{9}$$

$$\sum_{i \in \mathcal{M}_j} y_{ji} \le D_{jt}^{flex} \qquad \text{for all } j \in \mathcal{J} \tag{10}$$

$$x_i \ge 0 \qquad \text{for all } i \in \mathcal{I} \tag{11}$$

$$y_{ji}^a, y_{ji} \ge 0 \qquad \text{for all } j \in \mathcal{J} \text{ and } i \in \mathcal{M}_j \tag{12}$$

The objective function (6) maximizes the total revenue-to-go. The constraints (7) ensure that all accepted requests for product j are (temporarily) assigned to a specific execution mode $i \in \mathcal{M}_j$. The constraints (8) guarantee that the remaining capacity is sufficient for the capacity allocations to future requests, as well as for the existing commitments regarding flexible requests. The constraints (9) and (10) ensure that the allocations to future requests do not exceed the expected demand-to-come. Furthermore, the decision variables must be nonnegative (constraints (11) and (12)).

4 Value of flexibility

v_j : exact value of flexibility of product j	v_j^{DLP} : value of flexibility of product j derived from the DLP

Table 2: Additional notation introduced in Section 4

In this section, we show that flexible products possess an additional value that can be captured monetarily. We call this their "value of flexibility," which is analytically derived from the corresponding DP formulation and further investigated. We then turn to the DLP and show that this model completely fails to capture the value of flexibility because it ignores the possibility of substitution after the sale. This leads to a severe and systematic shortcoming of existing DLP-based approaches with regard to flexible products. Table 2 summarizes the additional notation introduced in this section.

4.1 Analytical derivation of the value of flexibility

Intuitively, it should often be preferable to sell a flexible product instead of a slightly more expensive and corresponding regular product; that is, to exchange a small amount of money for the opportunity to decide on the used resources at a later point in time. Let v_j denote the resulting value of flexibility of a certain product j; that is, the benefit the firm obtains from being able to postpone the assignment decision. Then, v_j can be obtained from comparing the revenue-to-go that the firm can achieve with the resources remaining after selling j with that achievable after selling a similar regular product, which only differs in its lack of supply-side flexibility. To obtain this regular product, we consider all execution modes of j and choose the one with the highest revenue-to-go after acceptance. Thus, we define:

$$v_j := V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y}^a + \boldsymbol{e}_j\right) - \max_{i \in \mathcal{M}_j} V_{t+1}\left(\boldsymbol{c} - \boldsymbol{a}_i, \boldsymbol{y}^a\right). \tag{13}$$

Applying the definition of opportunity cost (3) and (5), we obtain

$$\begin{aligned} v_j &:= V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y}^a + \boldsymbol{e}_j\right) - V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y}^a\right) + V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y}^a\right) - \max_{i \in \mathcal{M}_j} V_{t+1}\left(\boldsymbol{c} - \boldsymbol{a}_i, \boldsymbol{y}^a\right) \\ &= -\Delta_j^{flex} V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y}^a\right) - \max_{i \in \mathcal{M}_j}\left(V_{t+1}\left(\boldsymbol{c} - \boldsymbol{a}_i, \boldsymbol{y}^a\right) - V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y}^a\right)\right) \\ &= \min_{i \in \mathcal{M}_j} \Delta_i^{reg} V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y}^a\right) - \Delta_j^{flex} V_{t+1}\left(\boldsymbol{c}, \boldsymbol{y}^a\right). \end{aligned} \tag{14}$$

This shows that the value of flexibility v_j reflects the difference between the opportunity cost of an immediate assignment to the best possible execution mode and the true opportunity cost associated with the flexible product. Thus, the value of flexibility can also be interpreted as the additional amount of money a purchaser of a flexible product has to pay if he wants to be informed about the final execution mode immediately, so that the firm's expected total revenue remains unchanged. Moreover, $v_j \geq 0$ obviously holds, because all future acceptance decisions that are feasible in state $\left(\boldsymbol{c} - \boldsymbol{a}_{i'}, \boldsymbol{y}^a\right)$ with $i' = \arg\max_{i \in \mathcal{M}_j} V_{t+1}\left(\boldsymbol{c} - \boldsymbol{a}_i, \boldsymbol{y}^a\right)$ are also feasible in state $\left(\boldsymbol{c}, \boldsymbol{y}^a + \boldsymbol{e}_j\right)$.

Example 1: We consider a firm with two resources A and B, as well as an advanced selling process, such that the remaining capacity is only $c_A = c_B = 1$. The firm can offer one high priced and one low priced regular product in respect of each resource. The products $A1$ and $A2$ correspond to resource A, the products $B1$ and $B2$ to resource B. The prices are $r_{A1}^{reg} = r_{B1}^{reg} = \$1{,}000$ and $r_{A2}^{reg} = r_{B2}^{reg} = \$250$. In addition, the firm can sell a flexible product F at a price of $r_F^{flex} = \$200$. Regarding the demand forecast, it is assumed that, in the remaining booking horizon, there will be one request either for $A1$ or $B1$, each with a probability of 0.5. The firm's decision problem is now to decide on the acceptance of the current requests for the low priced products $A2$, $B2$, and F. Figure 1 illustrates the resulting optimal policy in terms of a decision tree. Circles represent the random nodes of the product requests and squares depict the firm's decision nodes.

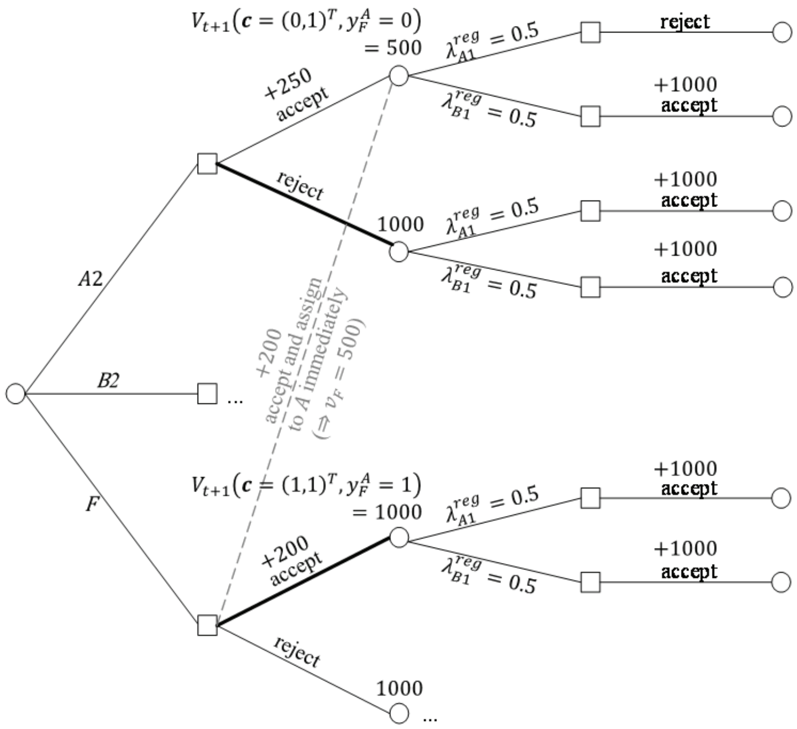

Figure 1: Illustration of optimal policy and value of flexibility

The upper part of the decision tree represents the arrival of a request for product $A2$. If this request is accepted, the firm earns a revenue of \$250 at the time of sale, and resource A's capacity is reduced accordingly. Given this decision, the firm can in future only accept the high-value request for $B1$, which has a 50% chance of occurring. Thus, the total expected revenue if the request for $A2$ is accepted, is \$750. On the other hand, if the firm rejects the request for $A2$, it will be able to accept any high-value request in the future, obtaining a guaranteed future revenue of \$1,000. Thus, it is optimal to reject

$A2$ and wait for a later request for $A1$ or $B1$. Following completely analogous considerations, it is also optimal to reject $B2$. Note that it is also not optimal to accept $A2$ and $B2$ simultaneously, as this would result in revenue of only \$500. In contrast, the acceptance of a flexible request (lower part of the decision tree) does not lead to an immediate reduction in capacity. In this case, it is possible to accept any future high-value request for $A1$ or $B1$ independently of the type, because one can reassign the request for F to the other free execution mode. The expected revenue consists of \$200 at the time of sale plus \$1,000 for the future request for $A1$ or $B1$. Overall, it is optimal to accept a flexible request and to reject corresponding low-value regular requests, even though the flexible request comes along with less immediate revenue. Now, if a customer of F wants to be informed about the execution mode immediately, the firm would lose \$500. More precisely, the firm would be in state $\left(\boldsymbol{c} = (0,1)^T, y_F^A = 0\right)$, instead of state $\left(\boldsymbol{c} = (1,1)^T, y_F^A = 1\right)$, as depicted by the dashed grey line in Figure 1. Of course, resource B would be equally suitable. Thus, the value of postponing the assignment is $v_F = V_{t+1}\left((1,1)^T, 1\right) - \max\left\{V_{t+1}\left((0,1)^T, 0\right), V_{t+1}\left((1,0)^T, 0\right)\right\} = \$1000 - \$500 = \$500$.

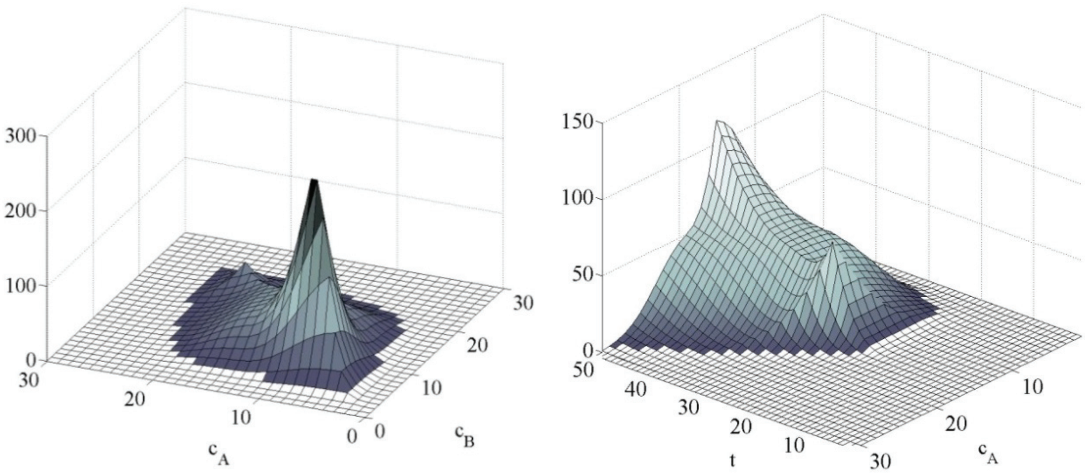

Figure 2: Illustration of the value of flexibility

Assuming the same resources and products as in Example 1, Figure 2 further illustrates that the value of flexibility v_F depends heavily on the current capacity situation, the point in time within the booking horizon, and the demand forecast. The left diagram of Figure 2 shows v_F subject to various levels of capacity c_A and c_B, 40 periods before departure, while assuming homogenous arrival probabilities of 0.2 for all products. The right diagram shows v_F subject to c_A and t from period 1 until departure at period

$T = 51$ with $c_B = 15$.The diagrams reflect a rather typical behavior of the value of flexibility, which was also observed in many other examples. However, even though desirable, there is no specific structure and no general monotonicity properties with respect to the dependencies that can be analytically derived, as counterexamples can always be constructed.

Overall, we conclude that flexible products come with an additional value of flexibility that can be captured monetarily. The value is always non-negative and postponing the resource allocation is therefore always valuable. However, the exact value is highly dynamic and depends strongly on the current parameters of the system's state.

4.2 Limitations of existing DLP-based approaches

Although the DLP presented in Section 3.3 is apparently designed for the integration of flexible products, the model is unable to adequately capture the value of flexibility. To show this, we rewrite the objective function V^{DLP} as a function of the right-hand side, in particular as a function of $\boldsymbol{c}$ and $\boldsymbol{y^a}$, i.e. $V^{DLP}\left(\boldsymbol{c},\, \boldsymbol{y^a}\right)$. Then, by replacing $V\left(\cdot\right)$ in (13) with the approximation $V^{DLP}\left(\cdot\right)$, we obtain the value of flexibility in the DLP:

$$v_j^{DLP} = V^{DLP}\left(\boldsymbol{c},\, \boldsymbol{y^a} + \boldsymbol{e}_j\right) - \max_{i \in \mathcal{M}_j} V^{DLP}\left(\boldsymbol{c} - \boldsymbol{a}_i,\, \boldsymbol{y^a}\right). \tag{15}$$

Now, note that both $V^{DLP}\left(\boldsymbol{c},\, \boldsymbol{y^a} + \boldsymbol{e}_j\right)$ and $V^{DLP}\left(\boldsymbol{c} - \boldsymbol{a}_{i'},\, \boldsymbol{y^a}\right)$ with $i' = \arg\max_{i \in \mathcal{M}_j} V^{DLP}\left(\boldsymbol{c} - \boldsymbol{a}_i,\, \boldsymbol{y^a}\right)$ have essentially the same primal solution in the optimum. An optimal solution of $V^{DLP}\left(\boldsymbol{c},\, \boldsymbol{y^a} + \boldsymbol{e}_j\right)$ can be obtained from an optimal solution of $V^{DLP}\left(\boldsymbol{c} - \boldsymbol{a}_{i'},\, \boldsymbol{y^a}\right)$ by simply assigning j to execution mode i' and leaving all the other decision variables unchanged. If it was better to assign j to another execution mode, then $i' = \arg\max_{i \in \mathcal{M}_j} V^{DLP}\left(\boldsymbol{c} - \boldsymbol{a}_i,\, \boldsymbol{y^a}\right)$ would be violated. As the objective functions contain only decision variables that are equal in both solutions, we have

$$V^{DLP}\left(\boldsymbol{c},\, \boldsymbol{y^a} + \boldsymbol{e}_j\right) = \max_{i \in \mathcal{M}_j} V^{DLP}\left(\boldsymbol{c} - \boldsymbol{a}_i,\, \boldsymbol{y^a}\right) \tag{16}$$

and, thus,

$$v_j^{DLP} = 0\,. \tag{17}$$

Loosely speaking, the DLP assumes that a flexible product simply occupies capacity in one of its execution modes, while its possible reassignment at a later point in time is simply beyond the scope of such a static model. In other words, a later reassignment is not necessary in the deterministic environment with its aggregated demands.

Consequently, DLP-based approaches completely ignore the value of flexibility inherent in flexible products and have a systematic bias against their acceptance. All products are only evaluated according to their at-time-of-sale revenue and their capacity consumption in a single “best” execution mode, always leading to a prioritized capacity allocation for comparable regular products. In the previous subsection, we have seen that the exact opposite might be optimal. This shortcoming hinders all existing control approaches based on the DLP, for example, booking limits obtained from the primal solution but also bid prices obtained from the dual solution, because the capacity allocations for flexible products are consistent with the bid price criterion (see Petrick et al. 2012). Therefore, the resulting bid prices do not reflect that the opportunity cost of flexible products might be considerably lower than that of regular products.

Example 2: We assume the same setting as in Example 1. Unlike the optimal policy, the primal solution of the corresponding DLP implies not reserving capacity for the flexible product, that is, $x_F = 0$, while there are positive allocations for all regular products ($x_{A1}, x_{A2}, x_{B1}, x_{B2} > 0$). Moreover, the obtained bid prices of the two capacity constraints are $\pi_A = \pi_B = \$250$. Thus, in a bid price control, only regular requests with immediate revenues of \$250 or higher are accepted if the capacity is sufficient. This leads to an acceptance decision regarding the low-value regular requests and a rejection decision regarding the flexible request, which is the exact opposite of the optimal policy.

5 Simulation-based optimization approach for controls based on the DLP approximation

$\tilde{v}_j$: (artificial) revenue markup of product j	$F^{const}_{\beta_{const}}(0)$: constant function to calculate $\tilde{v}_j$
$\boldsymbol{\beta}$: (arbitrary) parameters for simulation-based optimization	$F^{adapt}_{\beta_{const},\beta_{y^a},\beta_{time}}(z)$: dynamic function for $\tilde{v}_j$
z : (arbitrary) variables that are available during the booking horizon	π_h : bid price of resource h from DLP
$F_{\boldsymbol{\beta}}(z)$: (arbitrary) function to approximate $\tilde{v}_j$	$\tilde{\pi}_h$: bid price of resource h from DLP-inc

Table 3: Additional notation introduced in Section 5

To tackle the limitations of DLP-based approaches discussed in the previous section, we propose a generic approach that aims at incorporating an approximation of the value of flexibility into the DLP, using simulation-based optimization. Furthermore, we show how the new approach can be applied to bid price controls. Again, Table 3 provides an overview of the notation introduced in this section.

5.1 DLP with virtual revenues

We propose increasing the revenues of flexible products systematically and adequately. The resulting virtual revenues are then used within the DLP throughout the booking horizon. To formalize the approach, let $\tilde{v}_j$ denote the revenue markup of flexible product j. Substituting the revenue r_j^{flex} with the virtually increased revenue $r_j^{flex} + \tilde{v}_j$ in the DLP leads to the following modified model (DLP-inc):

$$V^{DLP-inc} = \max \sum_{i \in \mathcal{I}} r_i^{reg} \cdot x_i + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{M}_j} \left(r_j^{flex} + \tilde{v}_j \right) \cdot y_{ji} \quad (18)$$

subject to constraints (6)-(11) as in DLP.

From a theoretical perspective, we pick up on the analytical insight derived in respect of the DP in Section 4.1, namely that a flexible product with its lower opportunity cost can also be remodeled as a product with higher revenue but with immediate assignment to a certain execution mode. The resulting revenue markup equals the value of the flexibility that is sacrificed. By incorporating such a revenue markup into the DLP, it is now possible to place more value on a flexible product than on a comparable regular product. More specifically, the primal solution can now consist of capacity allocations for flexible products in a certain execution mode while, at the same time, the capacity allocations for comparable regular low-value products can be zero, even though these regular products are more expensive than the flexible ones. This is also reflected by the dual solution of DLP-inc.

Example 3: We assume the same setting as in Example 1, but increase the price of the flexible product F from \$200 to a virtual revenue greater than the revenue of the low-value regular products $A2$ and $B2$, for example, \$260. Using this virtual revenue in DLP-inc results in a primal solution of $x_{A1}, x_{B1}, x_F > 0$ and $x_{A2} = x_{B2} = 0$, as well as in bid prices of \$260 for both resources. These bid prices imply the acceptance of products

$A1$, $B1$, and F, as well as the rejection of products $A2$ and $B2$. Thus, the resulting bid price control is the same as the optimal policy.

Although clearly desirable, it is obviously not possible to compute the optimal revenue markup $\tilde{v}_j$ in a closed form for every single booking situation that might occur. More specifically, due to the given stochastic dynamic setting and the iterative solution of DLP instances over time, the calculation of optimal markups would suffer from the same curse of dimensionality as the exact DP. Therefore, we propose approximating $\tilde{v}_j$ by using a function $F_{\boldsymbol{\beta}}(\boldsymbol{z})$ of a vector of variables $\boldsymbol{z}$ that describe the current booking situation. The parameters $\boldsymbol{\beta}$ specify the influence of these variables on the revenue markup.

In the remainder of this paper, we consider two specific forms of the function $F_{\boldsymbol{\beta}}(\boldsymbol{z})$, which were identified as promising during numerical pretests:

- The simplest form is the constant function

  $$F_{\beta_{const}}^{const}(0) = \beta_{const}, \tag{19}$$

  which has only one parameter β_{const} and does not depend on any variables. With this function, the virtual revenue of a flexible product is either higher or lower than the revenue of corresponding regular products for the whole booking horizon.
- As demonstrated by Figure 2, the value of flexibility and, thus, the required markup might obviously change during the booking horizon. Thus, we also consider a function that allows $\tilde{v}_j$ to adapt according to the selling process. In line with the observations we made with regard to the value of flexibility in a number of analytical examples using the DP – similar to the one in Figure 2 – , we rely on the following two variables: To reflect the decisions made so far, we use the number of already accepted flexible requests y_j^a. Intuitively, the more flexible requests a capacity provider has accepted, the less the impact expected from an additional request's decision postponement is. As a second variable, we use t to indicate elapsed time. This suggests that capacity gradually becomes scarcer over time and the chances that a "fitting" regular request arrives in the remaining time decrease towards the end of the booking horizon. Besides β_{const}, the additional parameters β_{y^a} and β_{time} capture the influence of the two variables:

$$F^{adapt}_{\beta_{const},\beta_{y^a},\beta_{time}}(z) = \beta_{const} - \beta_{y^a} \cdot y^a_j + \beta_{time} \cdot t\ . \qquad (20)$$

To apply the approach, it is necessary to determine meaningful values for the parameters $\boldsymbol{\beta}$ of the function $F_{\boldsymbol{\beta}}(z)$. For this purpose, we propose the application of a standard method of iterative simulation-based optimization (see, e.g., Gosavi 2003 and Spall 2003) at the beginning of the booking horizon. In our context, an iteration of this approach consists of two steps and can be roughly summarized as follows: In the first step, values for the parameters $\boldsymbol{\beta}$ are passed to the simulation component to evaluate. For this evaluation, a set of n independent customer demand streams, each of which concerns the whole booking horizon, is used. This calibration set is generated in advance according to the decision maker's belief regarding possible realizations of the uncertain demand. Analogous to reality, a control approach incorporating $F_{\boldsymbol{\beta}}(z)$ is performed for each of these demand streams and the resulting total final revenue is stored. The revenues' average over all the streams of the calibration set is used to estimate the expected revenue of applying the approach with the current values for $\boldsymbol{\beta}$, and is passed to the optimization component. In the second step, the optimization component is used to calculate new $\boldsymbol{\beta}$ values by applying a standard metaheuristic optimization technique, which takes the current simulated average revenue and the data from previous iterations into account. The new values for $\boldsymbol{\beta}$ are then passed to the simulation component for evaluation, which starts the next iteration. As soon as a predefined convergence criterion is fulfilled, the optimization stops and the final configuration of parameter values $\boldsymbol{\beta}$ is obtained.

Having calibrated the parameters as described, the resulting function $F_{\boldsymbol{\beta}}(z)$ is actually applied in reality throughout the entire following booking horizon. The function parameters do not have to be recalibrated if the DLP is resolved within the booking horizon.

Finally, please note that during our numerical pretests, we also evaluated a number of more complex functional forms of $F_{\boldsymbol{\beta}}(z)$ (e.g. non-linear relations), as well as additional parameters to describe the booking situation. For example, we explicitly incorporated the expected demand, or the number of accepted regular products, i.e. the reserved capacity of each resource. However, complex functions with a high level of sophistication did not perform better than (19) and (20).

5.2 Application to static bid price controls

In general, the proposed approach is conceivable for any DLP-based capacity control approach. However, we restrict ourselves to its integration into the popular bid price controls, in which the shadow price of the corresponding capacity restriction (8) is directly used as a bid price π_h for each resource $h \in \mathcal{H}$. The total opportunity cost used in (2) and (4) is then approximated by the sum of the bid prices of all the required resources. Formally, given sufficient capacity, a request for a regular product i is accepted if and only if (see, e.g., Talluri and van Ryzin 1998)

$$r_i^{reg} \geq \sum_{h \in \mathcal{H}} a_{hi} \cdot \pi_h . \tag{21}$$

If there is a request for a flexible product j, profitability is checked against all possible execution modes; that is, it is accepted if and only if (see Petrick et al. 2012)

$$\exists i' \in \mathcal{M}_j \text{ with } r_j^{flex} \geq \sum_{h \in \mathcal{H}} a_{hi'} \cdot \pi_h . \tag{22}$$

In the case of acceptance of a regular request, the resources' remaining capacity $\boldsymbol{c}$ is immediately reduced by $\boldsymbol{a}_i$. If a flexible request is accepted, the commitment y_j^a is increased by one. Note that in order to determine whether there is sufficient capacity, one has to ensure that all accepted regular and flexible requests, as well as the incoming request, can be fulfilled. The literature has proposed a number of practical mechanisms that can be used for this purpose (see, e.g., Petrick et al. 2010).

Note that the standard bid price control described above even strengthens the discussed drawback of the DLP, because products are divided into two subsets: One group for which requests are accepted because the revenue is greater than or equal to the approximated opportunity cost, and another group for which the requests are rejected. Consequently, the bid price controls proposed so far have two systematic shortcomings: First, if the actual capacity utilization is high with respect to expected future demand, regular products are always preferred to cheaper flexible products, no matter how large their value of flexibility is. Second, if calculated bid prices are such that requests for flexible product j will be accepted, there is at least one execution mode $i' \in \mathcal{M}_j$ for which the flexible product's revenue will be greater than or equal to the sum of the affected resources' bid prices; that is, (22) holds. A comparison of (21) and (22) shows that regular requests that use the same resources as i' are then accepted as well. This acceptance is done first-come-first-served.

Now, in order to apply the artificial revenue markup to a bid price control, let $\tilde{\pi}_h$ denote the bid prices from DLP-inc. To formalize our approach, we straightforwardly modify the conditions (21) and (22): We simply use the bid prices obtained from DLP-inc for regular products and the bid prices, together with the virtual revenues, for flexible products. Thus, a request for a regular product i is accepted if and only if

$$r_i^{reg} \geq \sum_{h \in \mathcal{H}} a_{hi} \cdot \tilde{\pi}_h \tag{23}$$

and a request for a flexible product j is accepted if and only if

$$\exists i' \in \mathcal{M}_j \text{ with } r_j^{flex} + \tilde{v}_j \geq \sum_{h \in \mathcal{H}} a_{hi'} \cdot \tilde{\pi}_h \,. \tag{24}$$

Now, the bid prices obtained from the dual solution may be higher than the revenue of low-value regular products, preventing their acceptance. Flexible products, however, may be accepted during the booking horizon due to their artificially increased revenues.

6 Numerical experiments

In this section, we present the results of an extensive simulation study. The simulations were conducted on an Intel Xeon processor-based PC (Intel Core i7-2600 CPU, 4 Cores, 3.40 GHz, 8 GB RAM, Microsoft Windows 7 Enterprise SP1). All the algorithms were implemented in Matlab (Version 8, Release R2012b), using the Optimization Toolbox (in particular, the integrated linear programming routine *linprog*) and the Global Optimization Toolbox (in particular, the simulation-based optimization routine *patternsearch*), together with the Parallel Computing Toolbox.

In Section 6.1, we describe the basic simulation environment, i.e. the implemented capacity control mechanisms and the basic setting with its resource network structure and demand generation. In Section 6.2, we analyze the mechanisms' performance in detail. In Section 6.3, we extend the analysis in several directions to investigate the robustness of our findings. Here, we take a closer look at the influence of technical parameters governing the mechanisms, low-quality forecasts, and additional network structures.

6.1 Simulation environment

We consider the following mechanisms for capacity control:

- *BP* implements the classical bid price criterion. Bid prices obtained from model DLP are used in conditions (21) and (22) to decide on the acceptance of requests.
- *BP+Const* implements the first approach proposed in Section 5 and uses (19) to calculate a markup on the revenue of flexible products that remains constant throughout the selling horizon. The resulting virtual revenue is used in model DLP-inc to calculate bid prices, as well as in (23) and (24) to decide on the acceptance of requests. We use Matlab's routine *patternsearch* with a calibration set of $n = 50$ demand streams to determine the parameter β_{const} by means of simulation-based optimization.
- *BP+Adapt* is the second approach proposed in Section 5 and uses (20) in DLP-inc, as well as in (23) and (24). Again, we use *patternsearch* to calibrate the parameters β_{const}, β_{y^a} and β_{time}. Our choice of 50 demand streams for the calibration set is discussed in Section 6.3.
- *ExPost* calculates the optimal revenues, using perfect hindsight information about demand on a per-stream basis. As is common in revenue management simulation studies considering stochastic demands, we state the other methods' revenues relative to this benchmark because the DP (1) calculating the optimal expected value without perfect information is computationally intractable.

It is common practice in industry and academic research to resolve revenue management models at several points in time throughout the booking horizon in order to obtain updated bid prices that take the current capacity situation and demand forecast into account. This data is usually only available at certain points in time, for example, at major airlines, at around 10-15 pre-defined data collection points. Thus, we solve DLP and DLP-inc at 10 different points in time evenly distributed over the booking horizon. Alternative numbers of optimization are investigated in Section 6.3. To check whether the remaining capacity is sufficient for the acceptance of incoming requests, we use the pooling approach described in Petrick et al. (2010).

Next, we describe the basic setting used to analyze the performance of the control mechanisms. We consider a firm that can be thought of as an airline that offers the same flight at different times of day. The firm possesses four similar and parallel resources; that is, in the airline context, four flights that are operated per day, with a capacity of 200 each. The firm offers four regular products, which, in the airline context, correspond to ticket types or booking classes, on each resource. These products are priced at \$1,000, \$750, \$500, and \$250, respectively. In addition to the 16 regular products, a customer can buy a flexible product at a price of \$200 and is ensured of being assigned to one of the four resources later on. Note that this resource structure and the product definitions are fairly general and reflect many other examples in practice (see Section 2.2).

We generate the demand parameters for our simulations as follows: To vary the scarcity of capacity, we consider seven demand intensities, each given by a vector $[\alpha_i]_{m\times 1}$. Each α_i is multiplied by the corresponding resource i's capacity to obtain its total expected demand. For example, the vector $[1.0\ 1.0\ 1.0\ 1.0]$ implies that the expected demand equals the capacity and $[1.4\ 1.4\ 1.4\ 1.4]$ indicates that there is 40% excess demand. Subsequently, each resource's demand is split into demand for the associated products: 20% of the demand is for the flexible product and the remaining 80% is distributed among the four regular products at a ratio of 1:2:3:4 in the order of decreasing prices.

To obtain a discrete stochastic demand process, the selling horizon is divided into micro periods, in each of which at most one request arrives. The number of micro periods and the arrival probabilities are calculated such that the expected demand for each product sums up to the value given (see Subramanian et al. 1999).

We alternatively consider three arrival patterns regarding demand's temporal distribution. The first one, *low-before-high*, reflects the classical assumption that demand arrives in the order of non-increasing product prices. In the second, *time-homogenous* pattern, the arrival probabilities remain constant over the entire booking horizon. The third pattern, *mixed arrival*, is a mixture of the two aforementioned ones. In this pattern, cheaper products tend to be requested earlier than more expensive ones. Formally, the probability that a certain product will be requested at a certain point in time is simply the average of the probabilities that the same product will be requested at the same point in time in the low-before-high and time-homogeneous patterns.

Overall, we consider seven demand intensities, with three arrival patterns for the basic setting, resulting in 21 scenarios. For each scenario, we generate 200 customer streams, which are each used with all the control mechanisms. Independently of these evaluation streams, we generate 50 additional customer streams for each scenario that form the calibration set. Calibration took between 100 and 350 seconds for each scenario.

6.2 Numerical results for the basic setting

Table 4 shows the average revenue and the corresponding 99% confidence interval of the control mechanisms *BP*, *BP+Const*, and *BP+Adapt* relative to the *ExPost* revenue in all 21 scenarios. In addition, it shows the revenue gains of *BP+Const* and *BP+Adapt* over *BP*. For this purpose, we calculated the revenue difference together with the empirical standard deviation on a per-stream basis and conducted a standard paired t-test. If the 99% confidence interval of a gain does not include zero, the gain (or loss) is significant.

In general, Table 4 exhibits two well-known effects for all three control mechanisms. First, the more demand tends to arrive in a low-before-high order, the more challenging capacity control becomes, which is reflected by the revenues decreasing relative to the *ExPost* revenue. Second, capacity control becomes more important and challenging as the demand intensity increases. Whereas *BP+Const* cannot stop this deterioration of revenues for the low-before-high and time-homogeneous arrival patterns, it yields significantly higher revenues than *BP* in five of the seven scenarios for the mixed arrival pattern. These revenue gains range from 0.25% to 1.61%. *BP+Adapt* is more successful at keeping revenues at a constantly high level. Compared to *BP*, *BP+Const* and *BP+Adapt* only fails to show a revenue gain at a demand intensity of $[1.0\ 1.0\ 1.0\ 1.0]$. This is not surprising because – at least in expectation – capacity control is unnecessary for this demand intensity and *BP* therefore already achieves an extremely high revenue. For all other demand intensities, *BP+Adapt* delivers significantly higher revenues, and this gain tends to increase with demand intensity. The gain also depends on the arrival of demand: Whereas an average gain of 0.58% is attained across all seven demand intensities for the time-homogeneous arrival pattern, the average is 1.48% for the mixed arrival pattern and 3.13% for the low-before-high pattern. Here, the gain exceeds 4% considerably in four of the seven scenarios. However, in about half of the scenarios, *BP+Adapt* attains a gain of less than 1%. Note that even these seemingly small gains are considered important in revenue management as long as they are significant, because they can be attained without any relevant cost increase and directly add to profits, resulting in a much larger relative profit increase.

	Relative revenue			Relative gain over *BP*	
Scenario	*BP*	*BP+Const*	*BP+Adapt*	*BP+Const*	*BP+Adapt*
Low-before-high					
[1.0 1.0 1.0 1.0]	98.92 ± 0.26	98.92 ± 0.26	98.92 ± 0.26	0.00 ± 0.00	0.00 ± 0.00
[1.2 1.2 1.2 1.2]	92.72 ± 0.40	92.72 ± 0.40	97.98 ± 0.25	0.00 ± 0.00	5.26 ± 0.44
[1.4 1.4 1.4 1.4]	92.03 ± 0.38	92.03 ± 0.38	96.51 ± 0.32	0.00 ± 0.00	4.48 ± 0.38
[1.6 1.6 1.6 1.6]	91.62 ± 0.30	91.62 ± 0.30	96.52 ± 0.35	0.00 ± 0.00	4.89 ± 0.40
[1.5 1.3 1.1 0.9]	95.74 ± 0.38	95.74 ± 0.38	96.30 ± 0.38	0.00 ± 0.00	0.56 ± 0.26
[1.7 1.5 1.3 1.1]	91.76 ± 0.40	91.76 ± 0.40	94.18 ± 0.45	0.00 ± 0.00	2.42 ± 0.42
[1.7 1.6 1.5 1.4]	92.15 ± 0.37	92.15 ± 0.37	96.48 ± 0.31	0.00 ± 0.00	4.32 ± 0.44
Average	93.56	93.56	96.70	0.00	3.13
Mixed					
[1.0 1.0 1.0 1.0]	99.60 ± 0.11	99.60 ± 0.11	99.45 ± 0.14	0.00 ± 0.00	-0.15 ± 0.09
[1.2 1.2 1.2 1.2]	98.28 ± 0.18	98.28 ± 0.18	98.88 ± 0.15	0.00 ± 0.00	0.60 ± 0.18
[1.4 1.4 1.4 1.4]	96.08 ± 0.24	96.33 ± 0.19	97.80 ± 0.21	0.25 ± 0.23	1.72 ± 0.24
[1.6 1.6 1.6 1.6]	94.52 ± 0.29	96.13 ± 0.20	98.13 ± 0.16	1.61 ± 0.26	3.61 ± 0.30
[1.5 1.3 1.1 0.9]	96.87 ± 0.25	97.24 ± 0.19	97.50 ± 0.20	0.36 ± 0.22	0.63 ± 0.25
[1.7 1.5 1.3 1.1]	95.66 ± 0.29	96.55 ± 0.21	96.91 ± 0.19	0.89 ± 0.26	1.25 ± 0.26
[1.7 1.6 1.5 1.4]	94.90 ± 0.28	96.18 ± 0.19	97.58 ± 0.19	1.28 ± 0.26	2.68 ± 0.29
Average	96.56	97.19	98.04	0.63	1.48
Time-homogenous					
[1.0 1.0 1.0 1.0]	99.71 ± 0.07	99.71 ± 0.07	99.64 ± 0.09	0.00 ± 0.00	-0.07 ± 0.06
[1.2 1.2 1.2 1.2]	98.70 ± 0.12	98.70 ± 0.12	99.19 ± 0.10	0.00 ± 0.00	0.49 ± 0.15
[1.4 1.4 1.4 1.4]	97.52 ± 0.16	97.52 ± 0.16	98.46 ± 0.13	0.00 ± 0.00	0.94 ± 0.17
[1.6 1.6 1.6 1.6]	96.61 ± 0.18	96.61 ± 0.18	97.54 ± 0.15	0.00 ± 0.00	0.93 ± 0.15
[1.5 1.3 1.1 0.9]	97.97 ± 0.16	97.97 ± 0.16	98.29 ± 0.16	0.00 ± 0.00	0.32 ± 0.14
[1.7 1.5 1.3 1.1]	97.30 ± 0.19	97.30 ± 0.19	97.78 ± 0.15	0.00 ± 0.00	0.48 ± 0.20
[1.7 1.6 1.5 1.4]	96.99 ± 0.18	96.99 ± 0.18	97.92 ± 0.14	0.00 ± 0.00	0.94 ± 0.19
Average	97.83	97.83	98.40	0.00	0.58

Table 4: Revenues obtained with *BP*, *BP+Const*, and *BP+Adapt* and revenue gains of *BP+Const* and *BP+Adapt* (basic setting)

We also consider the average capacity utilization in percent (see Online Appendix A.1). It shows that, compared to *BP,* the number of flexible products sold when applying the new approaches clearly corresponds to their revenue gain. This number is almost identical for scenarios with no revenue gains, showing that the approaches indeed follow the same policy. In the case of revenue gains, a higher number of flexible products and a lower number of the cheapest regular products are usually sold. This shows that especially *BP+Adapt* is able to counteract the DLP's undervaluation of flexible products. However, for some scenarios with considerable revenue increases, we observe the opposite change in the number of accepted requests. We investigated this behavior in detail by analyzing the individual demand streams. Subsequently, we discuss the underlying effects using exemplary cutouts of demand streams.

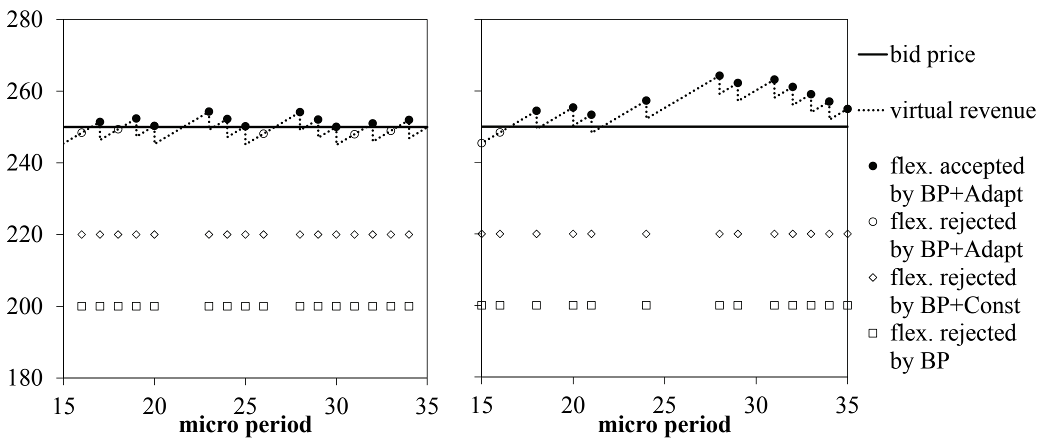

Figure 3: Exemplary cutouts of demand streams with low-before-high pattern and demand intensity $[1.6\ 1.6\ 1.6\ 1.6]$

Regarding the first case, *BP+Adapt* accepts more flexible requests than *BP* when demand is strong. To illustrate the results of our investigation, we use cutouts from two demand streams of a low-before-high arrival scenario with demand intensity $[1.6\ 1.6\ 1.6\ 1.6]$ (see Figure 3). As we consider an early point in time in the booking horizon (micro periods 15 to 35), the bid prices obtained from the optimization of DLP and DLP-inc at micro period 1, which are valid until the next optimization at micro period 128, are relevant. Demand is very high, so the DLP's solution contains only positive capacity allocations for regular products and the bid prices are equal to the price of the cheapest regular product, i.e. \$250. Thus, *BP* rejects requests for the flexible product priced at \$200. Requests for regular products do not arrive at this early stage in the booking process due to the low-before-high pattern. Regarding *BP+Const*, the simulation-based optimization determined a virtual revenue of about \$220 for the flexible product. Despite using this virtual revenue in DLP-inc, flexible products are not part of the primal solution and, again, a bid price of \$250 is obtained. Although a flexible request is now more valuable, it is still rejected. In *BP+Adapt*, DLP-inc is solved in micro period 1 with a virtual revenue of \$203.26 (not displayed here). Once more, this leads to bid prices of \$250 for each resource. As the virtual revenue is at first smaller than the bid price, requests for flexible products are initially rejected. However, the virtual revenue increases over time and exceeds \$250 in micro period 16. From then on, flexible requests are accepted. In the left hand cutout, a flexible request immediately arrives in micro period 16 and the virtual revenue, depending on the time and the number of accepted flexible requests, immediately falls below the bid price. Therefore, the next request (period 18) is rejected even though the virtual revenue again increases over time. In period 19, the virtual revenue again exceeds the bid price and a request is accepted. Thus, the virtual revenue declines again, etc. The right hand cutout shows a different demand stream. Here, by chance, fewer requests arrive and the flexible product's virtual revenue remains almost constantly above the bid prices. Overall, a considerable number of flexible products are sold, and, compared to the other approaches, the remaining capacity is considerably lower later on. This eventually leads to bid prices of above \$250 at a later reoptimization of DLP-inc and the rejection of cheap regular products, saving capacity for higher value products.

Figure 4 shows a cutout of a mixed arrival scenario with demand intensity $\left[1.4\ 1.4\ 1.4\ 1.4\right]$. It mainly differs from Figure 3's cutouts in that the bid prices of all three control mechanisms differ. Whereas the bid price used by *BP* is higher than \$200 and prevents the acceptance of flexible products, the bid price of *BP+Const* is equal to the virtual revenue of around \$280. Thus, *BP+Const* accepts all flexible requests. *BP+Adapt* has bid prices of almost \$260 and accepts only some requests for flexible products while rejecting others. Since we consider a mixed arrival pattern, requests for regular products also arrive between requests for flexible ones. The figure only displays requests for the cheapest products. While they are accepted according to the bid prices of *BP*, they are rejected by *BP+Const* and *BP+Adapt*.

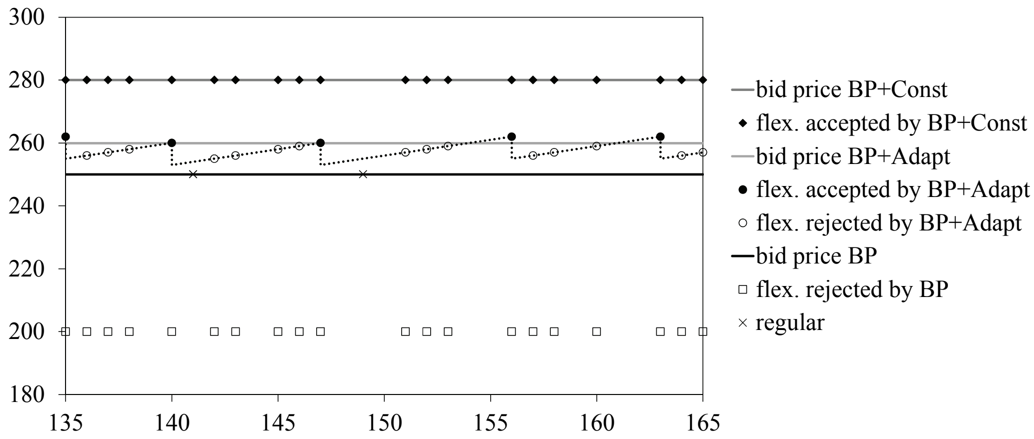

Figure 4: Exemplary cutout of demand stream with mixed arrival pattern and demand intensity $\begin{bmatrix}1.4 & 1.4 & 1.4 & 1.4\end{bmatrix}$

Both examples show how the new approaches substitute regular requests by flexible requests. Moreover, they illustrate that *BP+Const* can improve revenues; however it can only accept either all or none of the flexible requests between two reoptimizations.

Regarding the second case, where a decrease in the number of accepted flexible requests coincides with an increase in revenue, we observe that this happens in scenarios where demand slightly exceeds capacity. Here, DLP's primal solution envisions selling only very few flexible products, but the resulting low bid prices allow for the sale of flexible products until the next reoptimization. *BP+Adapt* successfully overcomes this drawback as it reduces the flexible product's virtual revenue after accepting a flexible request.

Because the performance of *BP+Adapt* is mostly superior to that of *BP+Const*, we concentrate on it in the following. To analyze the sensitivity of our results regarding the parameters of the basic setting, we considered a substantial number of variations that differ regarding prices, demand shares, as well as the number of resources and flexible products offered (see Online Appendix A.2). The major findings are mostly supported. Regarding the revenue improvements, the discussed structural relationships still apply. Naturally, the exact figures vary. There are small gains, but also impressive ones exceeding 10%. The gains are biggest when demand is low-before-high and smallest for time-homogeneous demand, where average gains almost never exceed 1%. Moreover, the flexible products' price seems to have no systematic influence on the gains, but with low-before-high demand, the revenue gains clearly increase in the demand share of flexible products. In contrast, with time-homogeneous demand, gains decrease in flexible products' share, and there is no obvious trend for mixed demand. The general revenue performance (not shown in the tables) is constantly good; that is, between 93.16 to 98.93% of *ExPost* in the worst scenarios. These are characterized by a high price difference between flexible and regular products, a low demand share of flexible products and the low-before-high arrival pattern. In the best scenarios, 98.45 to 99.76% of the *ExPost* revenue is achieved (at a demand ratio of regular products of 1:1:1:1).

6.3 Further investigations

In this subsection, we extend the analysis in three directions. First, we take a closer look at rather technical optimization parameters and justify the choice of ten optimizations throughout the booking horizon, as well as the use of 50 demand streams in the calibration set. Second, we investigate the influence of erroneous forecasts on the mechanisms' revenue performance because forecast quality is a major concern in practice. Third, we depart from parallel network structures and investigate two hub-and-spoke networks.

6.3.1 Variation of technical optimization parameters

Keep in mind that the considered capacity control approaches are static in the sense that the DLP is solved to obtain bid prices at only specific points in time throughout the booking horizon (Section 6.1). It is well known that the performance of such bid price controls can be sensitive to how often the underlying DLP model is solved and the bid prices are updated. Therefore, we conducted a number of tests to determine this influence and to choose an adequate number of optimizations. Using data from two typical scenarios of the basic setting, Figure 5 shows the average revenue subject to the number of optimizations. In line with the literature, a higher optimization frequency improves the revenue performance of both methods, but the improvement decreases with additional optimizations. In particular, we never observed substantial revenue gains above ten optimizations. However, the computational burden increases linearly. In view of this trade-off and typical optimization frequencies occurring in practice, ten optimizations seem reasonable. Moreover, our experiments indicate that *BP+Adapt* remains significantly superior to *BP* when more than ten optimizations are performed.

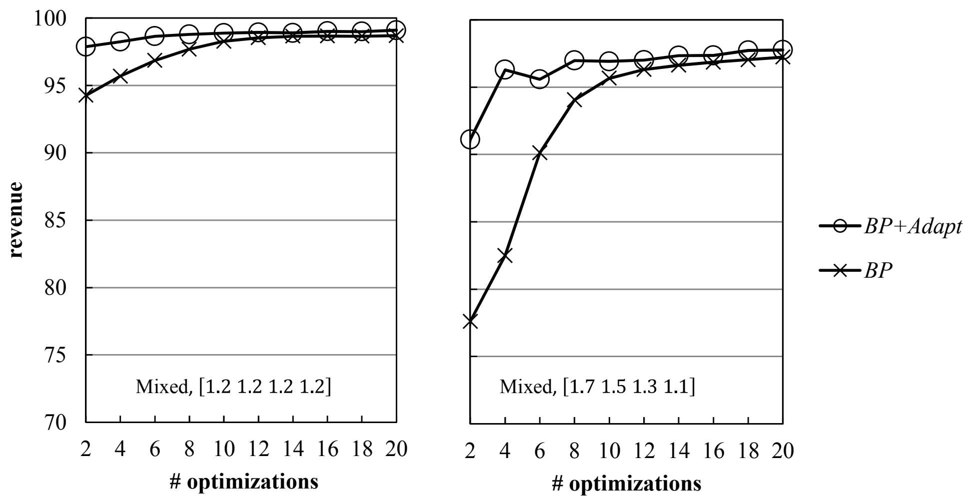

Figure 5: Revenue performance subject to number of optimizations

We performed similar tests to determine the size of the calibration sets. Figure 6 displays typical results obtained in three scenarios with 20 to 100 demand streams in the calibration set. As expected, it shows that the revenue performance suffers significantly if the calibration set is too small, but stays constant in most scenarios if at least 50 streams are used. The choice of 50 streams is also confirmed by another test in which we used a constant number of streams, but repeated the experiment with different random numbers; that is, different customer streams. While revenues fluctuate for small set sizes, revenue deviations are negligible for larger sets. For example, the pair-wise revenue differences observed using ten different calibration sets with 50 demand streams each were only around 0.1 to 0.3 percentage points.

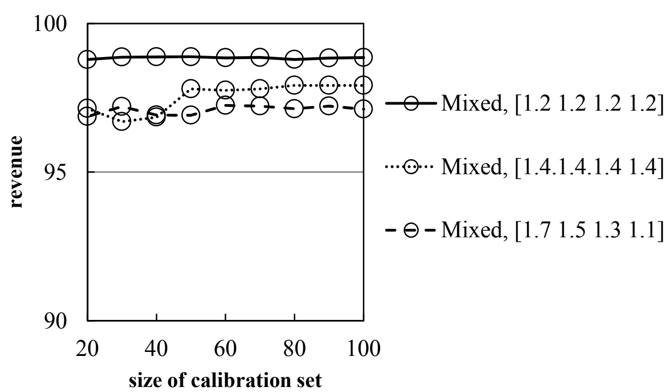

Figure 6: Revenue performance subject to size of calibration set

6.3.2 Forecast errors

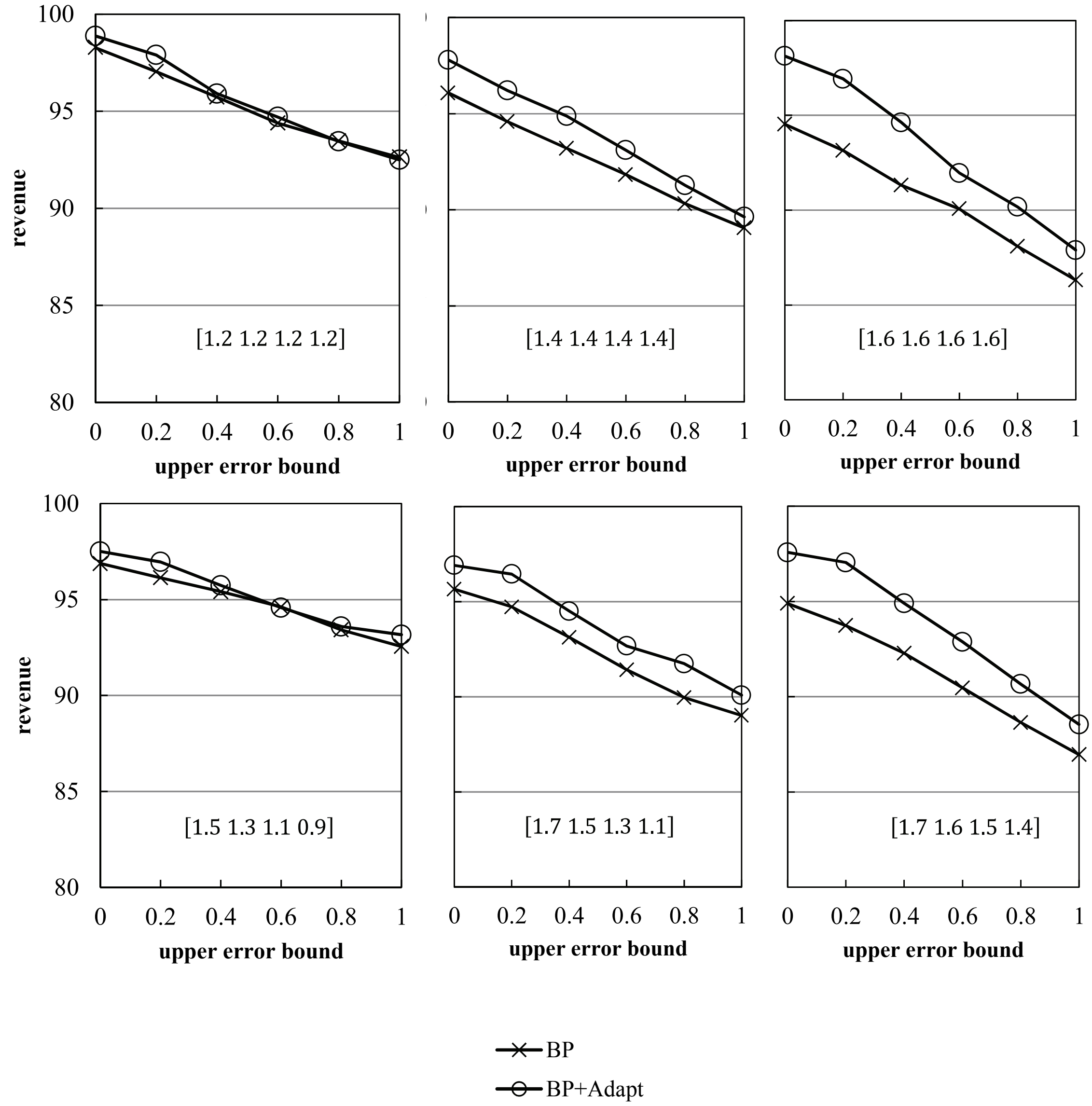

Figure 7: Revenue performance subject to forecast errors in mixed arrival setting

To consider forecast accuracy, we implemented the stochastic forecast error that we examined in our previous studies (Petrick et al. 2010, 2012). The forecast error is itinerary-based; that is, all demand forecasts concerning a specific itinerary and concerning the flexible product are disturbed by the same factor. The size of the forecast error is controlled by an upper error bound $\delta \in [0,1]$. A random number $\hat{\delta} \in U(-\delta,+\delta)$ is drawn within each simulation run for each of the basic setting's four itineraries and for the flexible product, disturbing the corresponding expected demand by the factor $(1+\hat{\delta})$. Moreover, the calibration sets used in the simulation-based optimization to parameterize the revenue markup are generated using this disturbed expected demand. Figure 7 illustrates the corresponding revenues in the mixed arrival setting. As expected, both approaches suffer from a worse demand forecast. Although there are settings for which the gain of *BP+Adapt* becomes smaller with decreasing forecast quality, this observation cannot be generalized. However, in all scenarios, the average revenue of *BP+Adapt* is never lower than that of *BP*.

6.3.3 Hub-and-spoke networks

Finally, we also consider two small hub-and-spoke networks. *H&S1* (see Figure 8) consists of three flight legs connecting the cities A, B, and C. There are two itineraries (A to B and A to C), each consisting of two flight legs. On each itinerary, we define four regular products, which are priced as in the basic setting. The flexible product at a price of $200 offers transportation from A to either B or C. Regarding the demand generation, we start with a total demand that equals the total capacity multiplied by a given demand intensity. Thereafter, this total demand is split into the product-specific demands as in the basic setting: 20% is for the flexible product, and 40% is divided among the regular products of each itinerary in a ratio of 1:2:3:4.

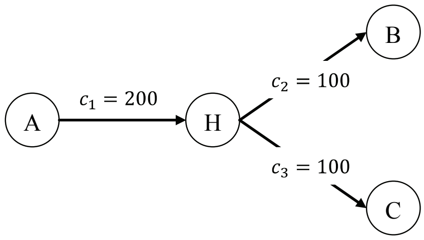

Figure 8: Hub-and-spoke network *H&S1*

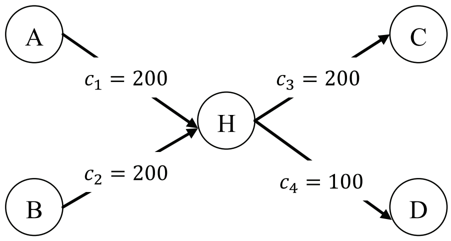

Figure 9: Hub-and-spoke network *H&S2*

H&S2 consists of four flight legs connecting the cities A, B, C, and D (see Figure 9). There are six itineraries: four single-leg itineraries from A or B to H, and from H to C or D, as well as the two-leg itineraries from A to C and from B to D. We define four regular products for each itinerary, priced from $500 to $100 for single-leg itineraries and from $1000 to $250 for connecting itineraries. There are two flexible products at a price of $90 each: F1 offers transportation from A or B to H, and F2 is from H to C or D. We again start with a total demand equal to the total capacity multiplied by a given demand intensity. Thereafter, 10% of the total demand is for F1 and F2, respectively. The remaining 80% is split into the regular products' demand. Overall, 30% of that remaining demand is for connecting itineraries and 70% is for single-leg itineraries.

Table 5 contains *BP+Adapt's* revenue relative to *ExPost* and revenue gain over *BP*. Again, the overall picture is familiar. The relative revenues are constantly high and *BP+Adapt's* advantage is biggest for low-before-high demand. There is a slight advantage for the mixed arrival pattern, and only a negligible advantage for the time-homogeneous arrival pattern. Comparing identical demand intensities and arrival patterns regarding *H&S1* and *H&S2*, we notice that the gain is almost always considerably higher in *H&S1*. This can be explained by *H&S1* containing only two-leg products and *H&S2* containing both single-leg and higher value two-leg products, but only single-leg flexible products, which are comparatively inferior to cheap two-leg products.

	Relative revenue *BP+Adapt*		Relative gain over *BP*	
Scenario	*H&S1*	*H&S2*	*H&S1*	*H&S2*
Low-before-high				
1.0	95.25 ± 0.19	97.84 ± 0.35	4.03 ± 0.73	0.34 ± 0.22
1.1	96.16 ± 0.30	97.80 ± 0.29	4.08 ± 0.63	4.06 ± 0.46
1.2	96.46 ± 0.48	96.83 ± 0.32	4.87 ± 0.65	2.08 ± 0.40
1.3	94.55 ± 0.40	94.83 ± 0.38	0.46 ± 0.64	0.26 ± 0.18
1.4	95.56 ± 0.57	93.06 ± 0.40	3.73 ± 0.66	0.72 ± 0.39
1.5	93.75 ± 0.65	94.15 ± 0.42	1.54 ± 0.57	2.32 ± 0.51
1.6	95.06 ± 0.54	94.21 ± 0.37	3.75 ± 0.63	2.46 ± 0.41
1.7	95.19 ± 0.51	95.09 ± 0.39	4.64 ± 0.71	4.43 ± 0.46
Average	95.25	95.48	3.39	2.08
Mixed				
1.0	96.05 ± 0.15	99.28 ± 0.14	1.44 ± 0.30	0.29 ± 0.12
1.1	97.03 ± 0.23	98.68 ± 0.15	2.13 ± 0.49	0.72 ± 0.22
1.2	97.61 ± 0.29	98.32 ± 0.20	2.28 ± 0.42	0.55 ± 0.18
1.3	96.97 ± 0.34	97.53 ± 0.23	0.39 ± 0.32	0.43 ± 0.15
1.4	96.85 ± 0.37	96.53 ± 0.27	1.74 ± 0.41	0.74 ± 0.19
1.5	96.08 ± 0.42	95.77 ± 0.28	1.99 ± 0.47	0.90 ± 0.23
1.6	95.71 ± 0.33	95.73 ± 0.30	2.01 ± 0.51	1.23 ± 0.25
1.7	95.98 ± 0.46	94.92 ± 0.31	2.59 ± 0.49	0.85 ± 0.23
Average	96.54	97.09	1.82	0.71
Time-homogenous				
1.0	96.20 ± 0.17	99.55 ± 0.08	0.31 ± 0.26	0.14 ± 0.11
1.1	97.28 ± 0.22	99.04 ± 0.10	0.60 ± 0.24	0.33 ± 0.12
1.2	98.26 ± 0.21	98.88 ± 0.11	0.25 ± 0.18	0.37 ± 0.13
1.3	97.96 ± 0.25	98.16 ± 0.16	0.46 ± 0.23	0.02 ± 0.13
1.4	97.63 ± 0.26	97.64 ± 0.16	0.49 ± 0.22	0.19 ± 0.11
1.5	97.24 ± 0.31	97.11 ± 0.18	0.82 ± 0.26	0.29 ± 0.14
1.6	97.00 ± 0.35	96.95 ± 0.19	0.99 ± 0.25	0.56 ± 0.13
1.7	96.73 ± 0.33	96.26 ± 0.19	0.95 ± 0.33	0.18 ± 0.12
Average	97.29	97.95	0.61	0.26

Table 5: Revenue gains of *BP+Adapt* (hub-and-spoke networks)

7 Conclusion

Having analyzed the numerical results in detail in the previous section, we now first take a broader perspective and discuss the assumptions made, as well as the methodology used. Finally, we summarize the main results and point out aspects worthy of future research.

7.1 Discussion

The assumptions made in this paper are in line with the standard traditional revenue management setting (see, e.g., Talluri and van Ryzin 2004a, Chapters 2 and 3). In particular, we focus on independent demand, as well as on DLP-based static bid price controls due to their high relevance in theory and practice. However, it is worth noting that recent theoretical work proposes generalizations. Choice-based revenue management addresses the first assumption and allows for more general customer behavior, but it is still rather theoretically challenging and not yet applicable to most real-world network revenue management settings. Moreover, in practice, customer choice behavior can currently be often incorporated by adjusting the input parameters, for example, by applying a fare transformation (Fiig at al. 2009 and Walczak et al. 2009). Moreover, our reasoning regarding the shortcoming of existing approaches also applies in principle to choice-based revenue management, where DLP-based approaches are also common (see, e.g., Liu and van Ryzin 2008 for the CDLP approach). The second assumption is addressed by research on dynamic bid prices and approximate dynamic programming approaches. These sophisticated control approaches are often interesting from a theoretical perspective, but are highly complex and only improve the overall revenue performance marginally in realistic settings. Thus, in most industries, these approaches have not yet been integrated into the revenue management software systems. Moreover, some of these approaches still rely partly on the DLP, for example, to divide the network problem into easier-to-calculate single-leg problems (see, e.g., Talluri and van Ryzin 2004a, Chapters 3.4.3 and 3.4.4). Our findings are also applicable to the well-known randomized linear program (RLP; see Talluri and van Ryzin 1999 for the standard formulation), which overcomes the DLP's limitation regarding expected values by repeatedly applying the

DLP to sample paths of demand in order to incorporate demand uncertainty. In short, our findings apply to all approaches that rely on static models and do not (fully) incorporate how decisions are made over time. From this perspective, our work is related to a current discussion in the stochastic programming community about what can go wrong when using deterministic optimization for a decision problem with uncertainty (see King and Wallace 2012).

Regarding the applied research methodology, we triangulate our subject from various perspectives. To investigate revenue management with flexible products in theory and practice, we conduct an extensive literature review, and – as literature from practice is scarce – provide anecdotal evidence and real-world examples. Then, following an analytical approach, we formally define the value of flexibility and argue that it is always zero in DLP-based approaches. Finally, we use simulation experiments to evaluate the performance of the proposed new capacity control approach and generalize these results to a certain extent. Note that, in general, an analytical solution of a problem would obviously be superior to a simulation-based evaluation. However, due to the absence of meaningful ways to analytically investigate bid price controls and DLP-based approaches, the standard way to evaluate these methods is to simulate customer streams in a number of real-world examples. Although such simulations are a widely used tool, they have important drawbacks (see, e.g., Law 2006, Chapter 1.9) and need to be handled with care. First, a single simulation run produces only a point estimate of the achievable total revenue. Second, when considering several simulation runs, one should not have too much confidence in the results. Third, the simulation results can only be as relevant as the considered scenarios are, and there is always the risk of observing artificial, scenario-specific effects. To meet the first two drawbacks, we make use of statistical methods. For each scenario, we perform 200 different simulation runs, each of which represents a complete customer stream. Based on this, we check the 99% confidence intervals of the total average revenue to estimate the accuracy of our results. To compare two control mechanisms, we use identical customer streams and report the average pair-wise gain (or loss) together with its confidence interval. Moreover, we question the performance of the considered controls and – at selected occasions – dig deeper into the data to see whether the mechanics leading to the observed results are traceable. To guard against the third drawback, we consider a wide range of scenarios based on realistic product and resource structures. Although the standard assumption in revenue management is that a perfect demand forecast is available, we also tested whether our results are robust if this assumption is violated, which often occurs in practice.

7.2 Summary and outlook

In this paper, we first consider the exact dynamic programming formulation for capacity control with flexible products and identify the value of flexibility. This monetary figure describes the value inherent in a flexible product's supply-side substitution possibilities. It accounts for the firm's ability to decide on the product's execution mode after uncertainty regarding demand has been resolved at the end of the booking horizon instead of at the time of sale. We have illustrated that the value of flexibility can be quite substantial compared to the product's prices, but does not exhibit any clear structures besides being nonnegative.

Thereafter, we show that the DLP approaches for capacity control with flexible products do not capture this value. This leads to the conclusion that the analytical and numerical studies on DLP-based approaches in the literature have consistently underestimated the performance of flexible products and might therefore have resulted in misleading recommendations regarding, for example, the introduction or configuration of such products.

In order to cope with the shortcomings discovered in existing approaches, we propose a new DLP-based approach. The basic idea is to systematically add a dynamic markup to the revenue of flexible products. This is accomplished by using a mathematical function that depends on properties of the current booking situation. Note that given "optimal" markups, our approach can in theory never perform worse than the existing approaches do. In practice, the parameters of the markup function can, for example, be calibrated using simulation-based optimization; we apply this method in the numerical experiments in this paper.

In our numerical study, we consider a wide range of scenarios based on product and resource structures which often occur in practice, either directly or as a substructure.

The new approach clearly outperforms existing control approaches that do not rely on virtual revenues. For example, we obtain revenue improvements up to 5% in many scenarios. Across all the settings considered, the arrival of demand has a strong impact on the new approaches' potential. The improvements tend to be considerably larger if more low value demand arrives before high value demand does, which is a demand behavior observed in most industries where revenue management is applied. When demand is time-homogeneous, improvements rarely exceed 1%, but are mostly still significantly positive. This confirms the theory and indicates that the way in which we calibrate the parameters of our approach by means of simulation-based optimization is reasonable. This in turn quantifies the abovementioned finding that existing studies consistently underestimate the benefits of flexible products.

Overall, the numerical results are quite promising and encourage future numerical studies on this topic. These could include applying the new approach to other DLP-based techniques prevalent in revenue management, such as the RLP (see Talluri and van Ryzin 1999), and to other settings with variants of the DLP, like the CDLP (see Liu and van Ryzin 2008) in a setting that incorporates customer choice behavior.

Finally, the results in this paper also have direct implications for future practical considerations, especially if low value demand tends to arrive before high value demand. As the idea of our new approach relies on a simple and straightforward markup argument, firms could easily incorporate the approach into their systems. Specifically, existing software routines for solving the DLP and determining acceptance decisions can be directly reused by simply modifying the input revenue parameters. The calibration of the markup function can be performed by a separate module. For example, in this paper, we used out-of-the-box simulation-based optimization procedures. Moreover, our numerical pretests showed that relatively simple and intuitive markup functions with only few parameters and variables perform quite well, as they do not tend to suffer from overfitting, and can easily be calibrated since good parameter values can be found within a reasonable time. This again supports practical applicability, as the function can be interpreted naturally and manual (re)calibration steps are easy to understand.

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Online Appendix

A.1 Average capacity utilization in the basic setting

Table A.1 shows the average capacity utilization of *BP*, *BP+Const*, and *BP+Adapt* in percent. The columns are labeled according to the products' prices (e.g. the columns labeled 200 refer to flexible products). For example, a number of 7.6 in column 1,000 means that, on average, over all 200 demand streams, 7.6% of the total capacity of 800 is used for products priced at $1,000.

	BP					*BP+Const*					*BP+Adapt*				
Scenarios	1,000	750	500	250	200	1,000	750	500	250	200	1,000	750	500	250	200
Low-before-high															
[1.0 1.0 1.0 1.0]	7.6	15.9	24.0	31.9	20.0	7.6	15.9	24.0	31.9	20.0	7.6	15.9	24.0	31.9	20.0
[1.2 1.2 1.2 1.2]	7.0	17.3	27.5	36.1	12.1	7.0	17.3	27.5	36.1	12.1	8.9	18.8	28.7	38.3	3.6
[1.4 1.4 1.4 1.4]	8.3	20.8	29.3	41.0	0.0	8.3	20.8	29.3	41.0	0.0	9.9	21.4	33.4	28.4	6.5
[1.6 1.6 1.6 1.6]	9.4	23.4	34.8	31.9	0.0	9.4	23.4	34.8	31.9	0.0	12.1	25.4	38.3	0.4	23.4
[1.5 1.3 1.1 0.9]	8.1	18.1	28.0	33.5	12.1	8.1	18.1	28.0	33.5	12.1	8.4	18.5	28.0	34.4	9.3
[1.7 1.5 1.3 1.1]	8.5	20.7	30.0	26.7	14.0	8.5	20.7	30.0	26.7	14.0	9.1	21.2	31.3	34.3	2.8
[1.7 1.6 1.5 1.4]	9.4	23.0	33.2	33.6	0.0	9.4	23.0	33.2	33.6	0.0	11.1	24.4	36.8	16.0	11.5
Mixed															
[1.0 1.0 1.0 1.0]	7.7	16.0	24.0	31.9	19.7	7.7	16.0	24.0	31.9	19.7	7.7	16.0	24.0	31.6	20.1
[1.2 1.2 1.2 1.2]	8.8	19.1	28.3	36.3	7.4	8.8	19.1	28.3	36.3	7.4	9.2	19.1	28.6	37.3	5.1
[1.4 1.4 1.4 1.4]	9.6	21.5	31.6	36.6	0.4	10.3	22.2	32.9	12.0	22.5	10.4	22.1	33.0	25.6	8.7
[1.6 1.6 1.6 1.6]	10.9	24.0	34.8	29.9	0.1	11.9	25.3	37.7	0.0	25.1	12.1	25.3	38.4	15.5	8.4
[1.5 1.3 1.1 0.9]	8.5	19.0	27.4	30.6	14.4	8.8	19.2	28.3	21.2	22.3	8.9	19.2	28.3	22.7	20.5
[1.7 1.5 1.3 1.1]	9.8	21.6	31.1	29.0	8.4	10.4	22.3	32.8	11.7	22.7	10.4	22.2	33.1	15.3	19.0
[1.7 1.6 1.5 1.4]	10.6	23.6	34.3	31.1	0.3	11.6	24.6	37.0	0.0	26.7	11.8	24.6	37.2	13.8	12.4
Time-homogenous															
[1.0 1.0 1.0 1.0]	7.9	15.8	23.9	31.9	19.8	7.9	15.8	23.9	31.9	19.8	7.9	15.8	23.9	31.7	20.0
[1.2 1.2 1.2 1.2]	9.3	18.7	28.2	37.1	6.7	9.3	18.7	28.2	37.1	6.7	9.5	18.9	28.6	37.7	4.4
[1.4 1.4 1.4 1.4]	10.7	21.4	31.7	35.7	0.5	10.7	21.4	31.7	35.7	0.5	11.0	21.9	33.0	29.6	3.8
[1.6 1.6 1.6 1.6]	12.1	24.2	35.3	28.4	0.0	12.1	24.2	35.3	28.4	0.0	12.2	24.6	36.5	25.4	1.2
[1.5 1.3 1.1 0.9]	9.3	18.6	27.4	32.3	12.3	9.3	18.6	27.4	32.3	12.3	9.4	18.8	27.8	33.2	10.0
[1.7 1.5 1.3 1.1]	10.8	21.5	31.3	30.8	5.5	10.8	21.5	31.3	30.8	5.5	11.0	22.0	32.6	20.9	12.9
[1.7 1.6 1.5 1.4]	11.8	23.7	34.5	29.8	0.2	11.8	23.7	34.5	29.8	0.2	12.0	24.5	36.5	16.7	9.7

Table A.1: Average capacity utilization according to product prices achieved by *BP*, *BP+Const*, and *BP+Adapt* (basic setting)

A.2 Variations of basic setting

We first systematically varied the prices and demand values given in the basic setting. Starting with a price of \$200, we consider the prices \$175 and \$225 for the flexible product. Likewise, in addition to the basic setting's 20% demand for the flexible product, we also consider a demand share of 10% and 30% for the flexible product. By combining prices and demand shares, we generate eight variations of the basic setting.

Second, we studied the impact of a different demand ratio for regular products, i.e. settings with a lower, as well as a higher, demand for the more expensive products. In particular, we assume that the demand for the four regular products follows the ratios 1:2:3:6, 1:3:6:10, 3:4:5:6, or 1:1:1:1 in decreasing price order.

Finally, we studied settings with a different number of resources and flexible products while maintaining the assumption of parallel resources. Therefore, we assume the same set of regular products and the demand ratios as in the basic setting and consider the following four variations:

- *R2F1* and *R6F1* consist of two and six resources, respectively, and one flexible product $F1$. We expect that 20% of the total demand is for $F1$.
- *R4F2* consists of four resources (A to D) and two flexible products $F1$ and $F2$, which are priced at $r_{F1}^{flex} = \$225$ and $r_{F2}^{flex} = \$175$, respectively. $F1$ ensures assignment to one of the resources from A to C and $F2$ ensures assignment to one of the resources from B to D. We assume that 10% and 20% of the total demand for the corresponding resources is for $F1$ and $F2$, respectively.
- *R4F3* consists of four resources (A to D) and three flexible products $F1$, $F2$, and $F3$, which are priced at \$225, \$200, and \$175, respectively. The execution modes of $F1$ are resources A and B, of $F2$ are resources B and C, and of $F3$ are resources C and D. We assume that 10%, 20%, and 30% of the total demand for the corresponding resources are for the products $F1$, $F2$, and $F3$, respectively.

The numerical analysis results in 16 tables analogous to Table 4 (Section 6.2), each of which contains the evaluation of 21 scenarios. For the sake of brevity, we report the revenue gains in percentage points achieved by *BP+Adapt* over *BP* with the corresponding 99% confidence intervals in Table A.2 and Table A.3.

	Variation of price and demand share of flexible product								Variation of demand ratio of regular products			
	175			200		225			1:2:3:6	1:3:6:10	3:4:5:6	1:1:1:1
Scenario	10%	20%	30%	10%	30%	10%	20%	30%				
Low-before-high												
[1.0 1.0 1.0 1.0]	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.15 ± 0.18	0.00 ± 0.00	0.00 ± 0.00	0.40 ± 0.29	0.23 ± 0.29	0.25 ± 0.26	0.25 ± 0.23
[1.2 1.2 1.2 1.2]	1.02 ± 0.27	5.71 ± 0.45	10.31 ± 0.77	0.99 ± 0.84	9.66 ± 0.76	1.56 ± 0.84	4.83 ± 0.44	9.04 ± 0.76	7.02 ± 0.48	9.65 ± 0.52	3.63 ± 0.77	8.68 ± 0.38
[1.4 1.4 1.4 1.4]	1.63 ± 0.39	4.18 ± 0.38	2.92 ± 0.88	2.25 ± 0.39	2.32 ± 0.88	2.87 ± 0.39	0.03 ± 0.45	1.74 ± 0.88	3.49 ± 0.53	4.11 ± 0.56	0.70 ± 0.38	5.49 ± 0.44
[1.6 1.6 1.6 1.6]	1.87 ± 0.43	4.75 ± 0.44	4.12 ± 0.56	2.53 ± 0.43	4.29 ± 0.62	3.18 ± 0.43	5.83 ± 0.44	4.48 ± 0.55	6.24 ± 0.53	7.42 ± 0.57	4.41 ± 0.35	8.54 ± 0.34
[1.5 1.3 1.1 0.9]	2.98 ± 0.46	0.69 ± 0.26	7.72 ± 0.75	2.49 ± 0.38	7.18 ± 0.75	2.19 ± 0.37	0.39 ± 0.19	6.63 ± 0.76	1.46 ± 0.38	1.80 ± 0.34	1.69 ± 0.35	1.66 ± 0.34
[1.7 1.5 1.3 1.1]	0.00 ± 0.00	2.94 ± 0.42	3.57 ± 0.77	0.00 ± 0.00	0.84 ± 0.92	0.00 ± 0.00	4.45 ± 0.47	2.25 ± 0.92	3.88 ± 0.52	6.43 ± 0.57	0.47 ± 0.51	4.82 ± 0.38
[1.7 1.6 1.5 1.4]	2.29 ± 0.39	3.79 ± 0.43	10.04 ± 0.61	2.51 ± 0.39	9.63 ± 0.61	2.74 ± 0.39	4.82 ± 0.43	11.64 ± 0.39	3.02 ± 0.63	3.93 ± 0.52	2.18 ± 0.38	4.84 ± 0.36
Average	1.40	3.15	5.53	1.54	4.85	1.81	2.91	5.11	3.65	4.80	1.90	4.89
Mixed												
[1.0 1.0 1.0 1.0]	0.06 ± 0.07	-0.17 ± 0.09	-0.03 ± 0.08	0.04 ± 0.07	-0.02 ± 0.07	0.03 ± 0.09	-0.12 ± 0.08	0.00 ± 0.07	0.10 ± 0.12	-0.12 ± 0.12	-0.05 ± 0.10	0.09 ± 0.08
[1.2 1.2 1.2 1.2]	0.45 ± 0.15	0.74 ± 0.17	2.00 ± 0.32	0.89 ± 0.19	1.64 ± 0.30	1.21 ± 0.18	0.44 ± 0.16	1.28 ± 0.27	0.73 ± 0.22	0.71 ± 0.23	0.47 ± 0.14	0.45 ± 0.13
[1.4 1.4 1.4 1.4]	0.76 ± 0.38	1.84 ± 0.23	1.30 ± 0.21	1.38 ± 0.38	1.07 ± 0.20	3.01 ± 0.32	2.05 ± 0.24	0.81 ± 0.20	1.72 ± 0.24	1.08 ± 0.29	1.99 ± 0.25	2.62 ± 0.22
[1.6 1.6 1.6 1.6]	1.33 ± 0.29	3.00 ± 0.30	1.35 ± 0.22	3.28 ± 0.30	1.62 ± 0.31	3.37 ± 0.27	4.01 ± 0.30	2.84 ± 0.32	2.56 ± 0.43	3.01 ± 0.45	3.31 ± 0.29	3.13 ± 0.24
[1.5 1.3 1.1 0.9]	0.21 ± 0.10	0.81 ± 0.16	1.46 ± 0.25	0.21 ± 0.10	1.16 ± 0.21	0.21 ± 0.13	1.05 ± 0.23	0.83 ± 0.21	0.83 ± 0.21	0.86 ± 0.27	0.55 ± 0.14	0.56 ± 0.18
[1.7 1.5 1.3 1.1]	0.92 ± 0.21	0.82 ± 0.18	0.71 ± 0.26	0.85 ± 0.20	0.96 ± 0.29	0.97 ± 0.21	2.02 ± 0.25	0.84 ± 0.26	1.57 ± 0.39	1.58 ± 0.35	1.49 ± 0.26	1.33 ± 0.24
[1.7 1.6 1.5 1.4]	1.06 ± 0.29	2.22 ± 0.28	1.69 ± 0.40	1.71 ± 0.29	2.26 ± 0.41	3.42 ± 0.26	3.03 ± 0.25	2.47 ± 0.41	1.60 ± 0.36	2.24 ± 0.37	2.42 ± 0.25	2.49 ± 0.21
Average	0.68	1.32	1.21	1.20	1.24	1.75	1.78	1.30	1.30	1.34	1.45	1.52
Time-homogenous												
[1.0 1.0 1.0 1.0]	0.00 ± 0.06	-0.08 ± 0.06	-0.01 ± 0.02	-0.03 ± 0.06	0.00 ± 0.02	-0.07 ± 0.07	-0.06 ± 0.05	0.00 ± 0.01	-0.03 ± 0.08	-0.04 ± 0.08	0.04 ± 0.08	0.08 ± 0.08
[1.2 1.2 1.2 1.2]	0.20 ± 0.08	0.61 ± 0.15	0.60 ± 0.20	0.39 ± 0.15	0.43 ± 0.17	0.76 ± 0.14	0.34 ± 0.14	0.12 ± 0.19	0.41 ± 0.14	0.47 ± 0.13	0.43 ± 0.13	0.36 ± 0.11
[1.4 1.4 1.4 1.4]	0.95 ± 0.20	0.94 ± 0.17	0.73 ± 0.13	1.27 ± 0.21	0.38 ± 0.17	1.74 ± 0.17	0.71 ± 0.20	0.40 ± 0.16	0.06 ± 0.19	0.19 ± 0.19	0.76 ± 0.16	1.21 ± 0.17
[1.6 1.6 1.6 1.6]	1.48 ± 0.23	1.23 ± 0.16	0.86 ± 0.21	1.80 ± 0.25	1.06 ± 0.20	2.30 ± 0.22	2.04 ± 0.21	1.15 ± 0.21	0.91 ± 0.23	1.01 ± 0.23	1.70 ± 0.21	1.79 ± 0.16
[1.5 1.3 1.1 0.9]	0.24 ± 0.10	0.46 ± 0.14	0.52 ± 0.18	0.17 ± 0.10	0.39 ± 0.17	0.19 ± 0.12	0.34 ± 0.17	0.41 ± 0.18	0.34 ± 0.13	0.42 ± 0.14	0.35 ± 0.13	0.23 ± 0.13
[1.7 1.5 1.3 1.1]	0.19 ± 0.08	0.31 ± 0.13	0.39 ± 0.20	0.27 ± 0.15	0.34 ± 0.22	0.73 ± 0.15	0.84 ± 0.19	0.80 ± 0.19	0.31 ± 0.22	0.34 ± 0.22	0.54 ± 0.17	0.40 ± 0.16
[1.7 1.6 1.5 1.4]	1.09 ± 0.21	0.64 ± 0.20	0.61 ± 0.22	1.37 ± 0.21	0.66 ± 0.21	1.92 ± 0.17	1.38 ± 0.21	0.91 ± 0.19	0.43 ± 0.23	0.52 ± 0.23	1.29 ± 0.17	1.17 ± 0.17
Average	0.59	0.59	0.53	0.75	0.46	1.08	0.80	0.54	0.35	0.42	0.73	0.75

Table A.2: Revenue gains of *BP+Adapt* (variation of price and demand share of flexible product and demand ratio of regular products)

Scenario	2R1F	**Scenario**	6R1F	**Scenario**	4R2F	4R3F
Low-before-high		**Low-before-high**		**Low-before-high**		
[1.0 1.0]	0.00 ± 0.00	[1.0 1.0 1.0 1.0 1.0 1.0]	0.01 ± 0.03	[1.0 1.0 1.0 1.0]	0.45 ± 0.24	0.57 ± 0.27
[1.2 1.2]	4.69 ± 0.60	[1.2 1.2 1.2 1.2 1.2 1.2]	5.51 ± 0.40	[1.2 1.2 1.2 1.2]	2.24 ± 0.42	2.60 ± 0.69
[1.4 1.4]	3.35 ± 0.67	[1.4 1.4 1.4 1.4 1.4 1.4]	4.83 ± 0.34	[1.4 1.4 1.4 1.4]	2.42 ± 0.41	4.89 ± 0.60
[1.6 1.6]	4.63 ± 0.38	[1.6 1.6 1.6 1.6 1.6 1.6]	5.59 ± 0.47	[1.6 1.6 1.6 1.6]	2.55 ± 0.40	2.43 ± 0.56
[1.1 0.9]	0.00 ± 0.00	[1.3 1.2 1.1 1.0 0.9 0.8]	2.55 ± 0.33	[1.5 1.3 1.1 0.9]	2.16 ± 0.44	1.20 ± 0.52
[1.3 1.1]	3.46 ± 0.60	[1.8 1.6 1.4 1.2 1.0 0.9]	2.31 ± 0.33	[1.7 1.5 1.3 1.1]	4.15 ± 0.43	4.95 ± 0.63
[1.5 1.2]	2.53 ± 0.62	[1.8 1.7 1.6 1.5 1.4 1.3]	3.48 ± 0.38	[1.7 1.6 1.5 1.4]	1.65 ± 0.35	6.47 ± 0.54
Average	2.67	Average	3.47	Average	2.23	3.30
Mixed		**Mixed**		**Mixed**		
[1.0 1.0]	0.03 ± 0.13	[1.0 1.0 1.0 1.0 1.0 1.0]	-0.13 ± 0.07	[1.0 1.0 1.0 1.0]	-0.20 ± 0.12	-0.26 ± 0.11
[1.2 1.2]	0.63 ± 0.19	[1.2 1.2 1.2 1.2 1.2 1.2]	0.57 ± 0.14	[1.2 1.2 1.2 1.2]	0.70 ± 0.15	0.31 ± 0.16
[1.4 1.4]	0.88 ± 0.24	[1.4 1.4 1.4 1.4 1.4 1.4]	1.36 ± 0.20	[1.4 1.4 1.4 1.4]	0.67 ± 0.24	0.77 ± 0.30
[1.6 1.6]	2.76 ± 0.34	[1.6 1.6 1.6 1.6 1.6 1.6]	3.77 ± 0.24	[1.6 1.6 1.6 1.6]	2.55 ± 0.31	1.66 ± 0.33
[1.1 0.9]	0.00 ± 0.12	[1.3 1.2 1.1 1.0 0.9 0.8]	0.09 ± 0.15	[1.5 1.3 1.1 0.9]	0.27 ± 0.14	0.18 ± 0.21
[1.3 1.1]	0.43 ± 0.21	[1.8 1.6 1.4 1.2 1.0 0.9]	0.82 ± 0.25	[1.7 1.5 1.3 1.1]	0.72 ± 0.17	0.52 ± 0.28
[1.5 1.2]	1.73 ± 0.42	[1.8 1.7 1.6 1.5 1.4 1.3]	2.13 ± 0.25	[1.7 1.6 1.5 1.4]	1.08 ± 0.26	1.38 ± 0.33
Average	1.08	Average	1.23	Average	0.83	0.65
Time-homogenous		**Time-homogenous**		**Time-homogenous**		
[1.0 1.0]	0.02 ± 0.10	[1.0 1.0 1.0 1.0 1.0 1.0]	-0.07 ± 0.05	[1.0 1.0 1.0 1.0]	-0.01 ± 0.06	0.01 ± 0.05
[1.2 1.2]	0.36 ± 0.17	[1.2 1.2 1.2 1.2 1.2 1.2]	0.50 ± 0.13	[1.2 1.2 1.2 1.2]	0.17 ± 0.10	0.27 ± 0.10
[1.4 1.4]	0.26 ± 0.24	[1.4 1.4 1.4 1.4 1.4 1.4]	0.13 ± 0.17	[1.4 1.4 1.4 1.4]	0.11 ± 0.10	-0.11 ± 0.17
[1.6 1.6]	1.17 ± 0.30	[1.6 1.6 1.6 1.6 1.6 1.6]	1.68 ± 0.17	[1.6 1.6 1.6 1.6]	1.00 ± 0.18	0.80 ± 0.16
[1.1 0.9]	0.06 ± 0.11	[1.3 1.2 1.1 1.0 0.9 0.8]	0.29 ± 0.12	[1.5 1.3 1.1 0.9]	0.05 ± 0.12	0.08 ± 0.13
[1.3 1.1]	0.28 ± 0.19	[1.8 1.6 1.4 1.2 1.0 0.9]	0.32 ± 0.11	[1.7 1.5 1.3 1.1]	0.33 ± 0.13	0.47 ± 0.15
[1.5 1.2]	0.38 ± 0.15	[1.8 1.7 1.6 1.5 1.4 1.3]	1.12 ± 0.17	[1.7 1.6 1.5 1.4]	0.63 ± 0.18	0.80 ± 0.19
Average	0.36	Average	0.57	Average	0.32	0.33

Table A.3: Revenue gains of *BP+Adapt* (variation of number of resources and flexible products)